South Carolina College- and Career-Ready Standards for Mathematics

Standards Unpacking Documents

Grade 6

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Standards Unpacking Documents – Grade 6

With the final approval of the *South Carolina College- and Career-Ready Standards for Mathematics* on March 11, 2015, educators were provided with clear, rigorous, and coherent standards for mathematics that would prepare students for success in their intended career paths that will either lead directly to the workforce or further education in post-secondary institutions. *South Carolina College- and Career-Ready Standards for Mathematics* contains South Carolina College- and Career-Ready (SCCCR) Content Standards for Mathematics that represent a balance of conceptual and procedural knowledge and specify the mathematics that students will master in each grade level and high school course.

The State Department of Education released Support Documents throughout the 2015-2016 school year to provide support for educators who are implementing the *South Carolina College- and Career-Ready Standards for Mathematics*. The Support Documents, which are organized by grades, are then organized by possible units of study which address all of the standards for that grade. The Support Documents can be found at http://ed.sc.gov/instruction/standards-learning/mathematics/support-documents-and-resources/. The purpose of these documents is to provide guidance as to how all the standards at each grade may be grouped into units. Since these documents are merely guidance, the State Department of Education encourages districts to implement the standards in a manner that best meets the needs of students.

To provide an additional supportive resource for South Carolina mathematics educators and continue to build upon the work of the State Department of Education, the South Carolina Leaders of Mathematics Education organization offered to create grade specific Standards Unpacking Documents. These documents would be organized by grade level and grouped by key concept. The *South Carolina College- and Career-Ready Standards for Mathematics* and the South Carolina grade specific Mathematics Support Documents as well as North Carolina and Kansas resources were utilized in the creation of the grade specific Standards Unpacking Documents. This document was adapted and modified specifically from the North Carolina Department of Education grade specific Mathematics Unpacked Content resources as well as the Kansas Association of Teachers of Mathematics Flip Books.

The Mathematics Standards Unpacking Documents were collaboratively written by South Carolina classroom teachers, instructional coaches, district leaders, and higher education faculty who are members of the South Carolina Leaders of Mathematics Education. It is with sincere appreciation that we humbly acknowledge the dedication, hard work and generosity of time provided by the members of the South Carolina Leaders of Mathematics Education who made the Mathematics Standards Unpacking Documents possible.

The primary purpose and goal of the Mathematics Standards Unpacking Documents are to assist and support educators who are teaching the *South Carolina College- and Career-Ready Standards for Mathematics* and to increase student achievement by ensuring educators understand specifically what the standards mean a student must know, understand and be able to do. These documents may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. These documents, along with on-going professional development, may be one of many resources used to understand and teach *South Carolina College- and Career-Ready Standards for Mathematics*.

South Carolina College- and Career-Ready Standards for Mathematics
Mathematical Process Standards

The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Standards for Mathematics for each grade level and course. Since the Process Standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the Content Standards, the Process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

Students who are college- and career-ready take a productive and confident approach to mathematics. They are able to recognize that mathematics is achievable, sensible, useful, doable, and worthwhile. They also perceive themselves as effective learners and practitioners of mathematics and understand that a consistent effort in learning mathematics is beneficial.

The Program for International Student Assessment defines mathematical literacy as "an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens" (Organization for Economic Cooperation and Development, 2012).

A mathematically literate student can:

1. **Make sense of problems and persevere in solving them.**
    a. Relate a problem to prior knowledge.
    b. Recognize there may be multiple entry points to a problem and more than one path to a solution.
    c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem.
    d. Evaluate the success of an approach to solve a problem and refine it if necessary.

2. **Reason both contextually and abstractly.**
    a. Make sense of quantities and their relationships in mathematical and real-world situations.
    b. Describe a given situation using multiple mathematical representations.
    c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation.
    d. Connect the meaning of mathematical operations to the context of a given situation.

3. **Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others.**
    a. Construct and justify a solution to a problem.
    b. Compare and discuss the validity of various reasoning strategies.
    c. Make conjectures and explore their validity.
    d. Reflect on and provide thoughtful responses to the reasoning of others.

4. **Connect mathematical ideas and real-world situations through modeling.**
    a. Identify relevant quantities and develop a model to describe their relationships.
    b. Interpret mathematical models in the context of the situation.
    c. Make assumptions and estimates to simplify complicated situations.
    d. Evaluate the reasonableness of a model and refine if necessary.

5. **Use a variety of mathematical tools effectively and strategically.**
    a. Select and use appropriate tools when solving a mathematical problem.
    b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.

6. **Communicate mathematically and approach mathematical situations with precision.**
    a. Express numerical answers with the degree of precision appropriate for the context of a situation.
    b. Represent numbers in an appropriate form according to the context of the situation.
    c. Use appropriate and precise mathematical language.
    d. Use appropriate units, scales, and labels.

7. **Identify and utilize structure and patterns.**
    a. Recognize complex mathematical objects as being composed of more than one simple object.
    b. Recognize mathematical repetition in order to make generalizations.
    c. Look for structures to interpret meaning and develop solution strategies.

Number System 6.NS

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision are: reciprocal, inverse, greatest common factor, least common multiple, prime factorization, distributive property, rational number, integers, quadrant, opposite, zero pair, additive inverse, absolute value, and inequality (including ≠).

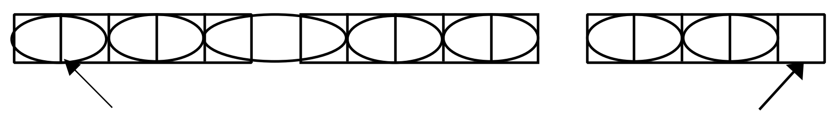
SCCCR Mathematics Standard	Unpacking What do these standards mean a child will know and be able to do?
6. NS.1 Compute and represent quotients of positive fractions using a variety of procedures (e.g., visual models, equations, and real-world situations).	In 5th grade students divided whole numbers by unit fractions and divided unit fractions by whole numbers. Students continue to develop this concept by using visual models and equations to divide whole numbers by fractions and fractions by fractions to solve real-world situations. Students develop an understanding of the relationship between multiplication and division. (5.NSF.7)

Clarifying Notes

- Divide positive fractions by fractions using visual models and equations.
- Solve real-world problems using division of fractions

Example 1:

Students understand that a division problem such as $3 \div \frac{2}{5}$ is asking, "how many $\frac{2}{5}$ are in 3?" One possible visual model would begin with three whole and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of ½. Therefore, $3 \div \frac{2}{5} = 7\frac{1}{2}$, meaning there are 7½ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves.



These two sections in the oval represent 1 group of two-fifths. This section represents one-half of two-fifths

Students also write contextual problems for fraction division problems. For example, the problem $\frac{2}{3} \div \frac{1}{6}$ can be illustrated with the following word problem:

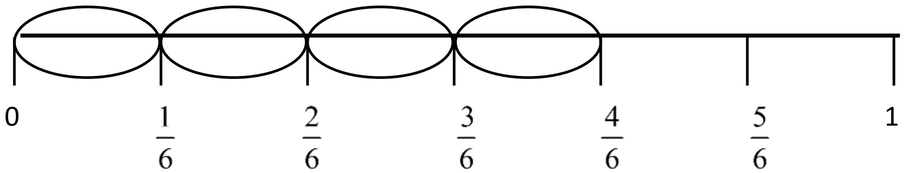
Example 2:

Susan has $\frac{2}{3}$ of an hour left to make cards. It takes her about $\frac{1}{6}$ of an hour to make each card. About how many can she make?

This problem can be modeled using a number line.

a. Start with a number line divided into thirds.

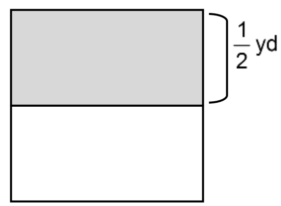
b. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.

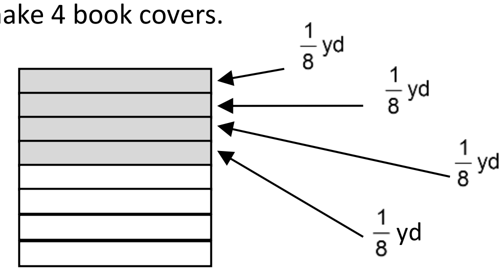


c. Each circled part represents ⅙. There are four sixths in two-thirds; therefore, Susan can make 4 cards.

Example 3:

Michael has $\frac{1}{2}$ of a yard of fabric to make book covers. Each book cover is made from $\frac{1}{8}$ of a yard of fabric. How many book covers can Michael make? Solution: Michael can make 4 book covers.





Example 4:

Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution.

Context: A recipe requires $\frac{2}{3}$ of a cup of yogurt. Rachel has $\frac{1}{2}$ of a cup of yogurt from a snack pack. How much of the recipe can Rachel make?

Explanation of Model:

The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show the $\frac{1}{2}$ cup.

The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally.

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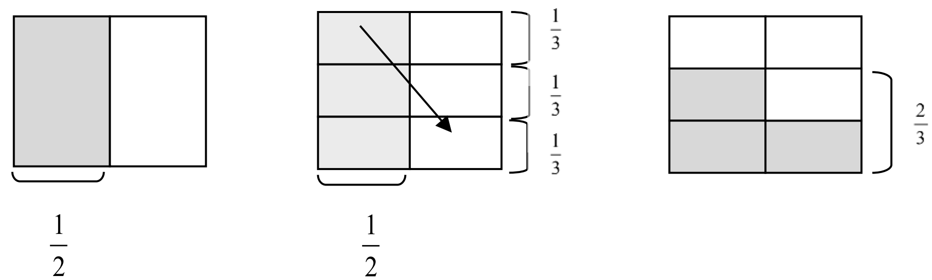
The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model.

$\frac{2}{3}$ is the new referent unit (whole).

3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so only ¾ of the recipe can be made.



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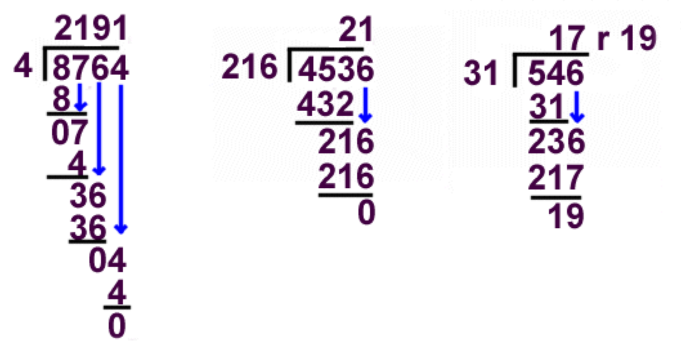
<td>6.NS.2 Fluently divide multi-digit whole numbers using a standard algorithmic approach.</td>
<td>
In the elementary grades, students were introduced to division through concrete models and various strategies to develop an understanding of this mathematical operation (limited to 4-digit numbers divided by 2-digit numbers). In 6^{th} grade, students become fluent in the use of a standard division algorithm, continuing to use their understanding of place value to describe what they are doing. This should involve numbers with more than 4 digits to show fluency. Place value has been a major emphasis in the elementary standards.

<u>Clarifying Notes</u>

Divide whole numbers by whole numbers
Convert remainders to fractional parts in simplest form and decimal notation

<u>Example 1:</u>


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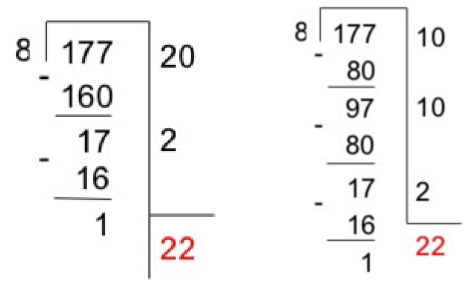
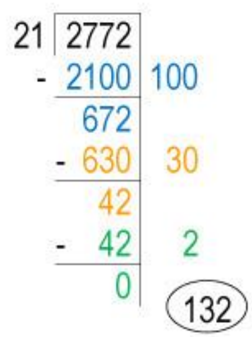
<u>Example 2:</u>

The method to the right allows more opportunities to help students struggling with larger math facts.

<u>Example 3:</u>

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<td>6.NS.3 Fluently add, subtract, multiply and divide multi-digit decimal numbers using a standard algorithmic approach.</td>
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Fluency is built upon a student’s ability to be flexible, accurate and efficient when computing. In 4th and 5th grades, students added and subtracted decimals to the hundredths place using concrete area models and drawings. Multiplication and division of decimals using concrete area models and drawings were introduced in 5th grade (decimals to the hundredths place). In 6th grade, students become fluent in the use of a standard algorithm of each of these operations.

<u>Clarifying Notes</u>

Perform all operations with decimal notation
Modeling equivalent numerical expressions to support the understanding of division of decimal numbers

The use of estimation strategies supports student understanding of decimal operations.
Use the fact that 13 × 17 = 221 to find the following.
a) 13 × 1.7
b) 130 × 17
c) 13 × 1,700
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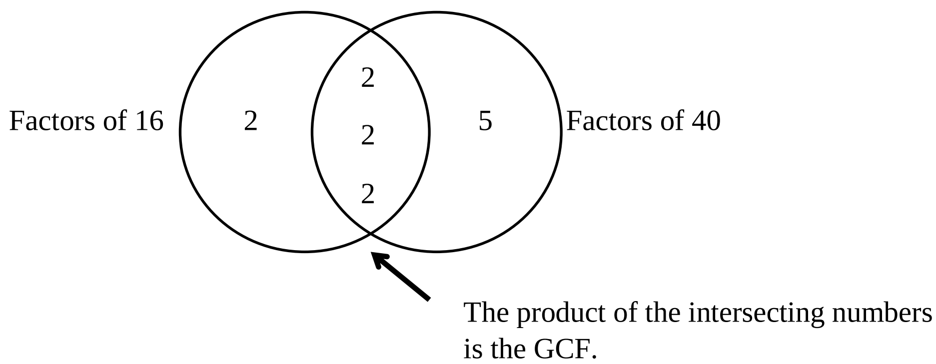
	d) 1.3×1.7 e) $2{,}210 \div 13$ f) $22{,}100 \div 17$ g) $221 \div 1.3$ All these solutions use the associative and commutative properties of multiplication (explicitly or implicitly). a) $13 \times 1.7 = 13 \times (17 \times 0.1) = (13 \times 17) \times 0.1$, so the product is one-tenth the product of 13 and 17. In other words, $13 \times 1.7 = 22.1$. b) Since one of the factors is ten times one of the factors in 13×17, the product will be ten times as large as well: $130 \times 17 = 2{,}210$. c) $13 \times 1{,}700 = 13 \times (17 \times 100) = (13 \times 17) \times 100$, so $13 \times 1{,}700 = 22{,}100$. d) Since each of the factors is one tenth the corresponding factor in 13×17, the product will be one one-hundredth as large: $1.3 \times 1.7 = 2.21$. e) $2{,}210 \div 13 = ?$ is equivalent to $13 \times ? = 2{,}210$. Since the product is ten times as big and one of the factors is the same, the other factor must be ten times as big. So, $2{,}210 \div 13 = 170$. f) As in the previous problem, the product is 100 times as big, and since one factor is the same, the other factor must be 100 times as big: $22{,}100 \div 17 = 1{,}300$. g) $221 \div 1.3 = ?$ is equivalent to $1.3 \times ? = 221$. Since the product is the same size and one of the factors is one-tenth the size, the other factor must be ten times as big. So, $221 \div 1.3 = 170$.
6.NS.4 Find common factors and multiples using two whole numbers. a. Compute the greatest common factor (GCF) of two numbers both less than or equal to 100. b. Compute the least common multiple (LCM) of two numbers both less than or equal to 12. c. Express sums of two whole numbers, each less than or equal to 100, using the distributive property to factor out a common factor of the original addends.	In elementary school, students identified primes, composites and factor pairs (4.OA.4). In 6th grade, students will find the greatest common factor of two whole numbers less than or equal to 100. **<u>Clarifying Notes:</u>** • Compute the greatest common factor (GCF) of two numbers both less than or equal to 100. • Compute the least common multiple (LCM) of two numbers both less than or equal to 12. • Express sums of two whole numbers, each less than or equal to 100, using the distributive property to factor out a common factor of the original addends. • Understand that greatest common factor and least common multiple are ways to discuss number relationships in multiplication and division. • Understand the process of prime factorization. • Understand the distributive property using sums and its use in adding numbers 1-100 with a common factor. • Use LCM and GCF to teach fluency for adding and subtracting of fractions using a standard algorithmic approach. For example, the greatest common factor of 40 and 16 can be found by 1) listing the factors of 40 (1, 2, 4, 5, 8, 10, 20, 40) and 16 (1, 2, 4, 8, 16), then taking the greatest common factor (8). Eight (8) is also the largest number such that the other factors are **relatively prime** (two

numbers with no common factors other than one). For example, 8 would be multiplied by 5 to get 40; 8 would be multiplied by 2 to get 16. Since the 5 and 2 are relatively prime, then 8 is the greatest common factor. If students think 4 is the greatest, then show that 4 would be multiplied by 10 to get 40, while 16 would be 4 times 4. Since the 10 and 4 are not relatively prime (have 2 in common), the 4 cannot be the greatest common factor.

2) listing the prime factors of 40 (2 • 2 • 2 • 5) and 16 (2 • 2 • 2 • 2) and then multiplying the common factors
(2 • 2 • 2 = 8).



Another approach is doing GCF and LCM by division of primes.

2	8,12
2	4,6
	2, 3

Start dividing by the smallest prime number that divides evenly into all values.
Once you cannot divide both values by the same prime, the two prime divisors when multiplied give you the GCF.

2	8,12
2	4,6
2	2, 3
3	1, 3
	1, 1

Begin by dividing all values by the smallest prime that divide into either value evenly.
Continue until all values have been divided and the remainders for each value are 1.
Multiply all the prime divisors together to calculate the LCM.

Students also understand that the greatest common factor of two prime numbers is 1.

<u>Example 1:</u>

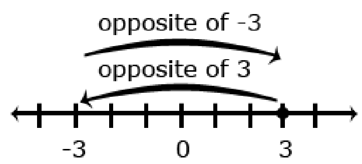
Use the greatest common factor and the distributive property to find the sum of 36 and 8.

36 + 8 = 4 (9) + 4(2)
44 = 4 (9 + 2)
44 = 4 (11)
44 = 44✓

	<u>Example 2:</u> Ms. Spain and Mr. France have donated a total of 90 hot dogs and 72 bags of chips for the class picnic. Each student will receive the same amount of refreshments. All refreshments must be used. a. What is the greatest number of students that can attend the picnic? b. How many bags of chips will each student receive? c. How many hotdogs will each student receive? *Solution:* a. Eighteen (18) is the greatest number of students that can attend the picnic (GCF). b. Each student would receive 4 bags of chips. c. Each student would receive 5 hot dogs.
6.NS.5 Understand that the positive and negative representations of a number are opposites in direction and value. Use integers to represent quantities in real-world situations and explain the meaning of zero in each situation.	Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electrical charge). **<u>Clarifying Notes:</u>** • Include temperature, elevation, credits/debits • Include vertical and horizontal number lines <u>Example:</u> a. Use an integer to represent 25 feet below sea level b. Use an integer to represent 25 feet above sea level. c. What would 0 (zero) represent in the scenario above? *Solution:* a. -25 b. +25 c. 0 would represent sea level
6.NS.6 Extend the understanding of the number line to include all rational numbers and apply this concept to the coordinate plane. a. Understand the concept of opposite numbers, including zero, and their relative locations on the number line.	In elementary school, students worked with positive fractions, decimals and whole numbers on the number line and in Quadrant I of the coordinate plane. In 6^{th} grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e. thermometer) which facilitates the movement from number lines to coordinate grids. **<u>Clarifying Notes:</u>** • Include vertical and horizontal number lines, plot all rational numbers. • Understand the effects of reflections on the ordered pairs (e.g., the ordered pair (2, 3) reflected about the y-axis becomes (-2, 3)).

b. Understand that the signs of the coordinates in ordered pairs indicate their location on an axis or in a quadrant on the coordinate plane.
c. Recognize when ordered pairs are reflections of each other on the coordinate plane across one axis, both axes, or the origin.
d. Plot rational numbers on number lines and ordered pairs on coordinate planes.

Students recognize that a number and its opposite are an equal distance from zero. The opposite sign (–) moves the number to the opposite side of 0. For example, – 4 could be read as “the opposite of 4” which would be negative 4. In the example, – (–6.4) would be read as “the opposite of the opposite of 6.4” which would be 6.4. Zero is its own opposite.



Example 1:

What is the opposite of $2\frac{1}{2}$? Explain your answer?

Solution:

$-2\frac{1}{2}$ because it is the same distance from 0 on the opposite side.

Example 2:

Order the numbers –4.5, 2, 3.2, $-3\frac{3}{5}$, 0.2, –2, $\frac{11}{2}$ from least to greatest. Justify your reasoning.

Solution:
The numbers in order from least to greatest are:
–4.5, $-3\frac{3}{5}$, –2, 0.2, 2, 3.2, $\frac{11}{2}$
Students place each of these numbers on a number line to justify this order.

Students worked with Quadrant I in elementary school. As the *x*-axis and *y*-axis are extended to include negatives, students begin to with the Cartesian Coordinate system. Students recognize the point where the *x*-axis and *y*-axis intersect as the origin. Students identify the four quadrants and are able to identify the quadrant for an ordered pair based on the signs of the coordinates. For example, students recognize that in Quadrant II, the signs of all ordered pairs would be (–, +).

Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs (-2, 4) and (-2, -4), the *y*-coordinates differ only by signs, which represents a reflection across the *x*-axis. A change in the *x*-coordinates from (-2, 4) to (2, 4), represents a

	reflection across the *y*-axis. When the signs of both coordinates change, (2, -4) changes to (-2, 4), the ordered pair has been reflected across both axes. <u>Example:</u> Graph the following points in the correct quadrant of the coordinate plane. If the point is reflected across the *x*-axis, what are the coordinates of the reflected points? What similarities are between coordinates of the original point and the reflected point? $\left(\frac{1}{2}, -3\frac{1}{2}\right)\left(-\frac{1}{2}, -3\right)$ *Solution:* The coordinates of the reflected points across the x-axis would be $\left(\frac{1}{2},\ 3\frac{1}{2}\right)\left(-\frac{1}{2},\ 3\right)$. Note that the *y*-coordinates are opposites of the original y-coordinates.
6.NS.7 Understand and apply the concepts of comparing, ordering, and finding absolute value to rational numbers. a. Interpret statements using equal to (=) and not equal to (≠). b. Interpret statements using less than (<), greater than (>), and equal to (=) as relative locations on the number line. c. Use concepts of equality and inequality to write and to explain real-world and mathematical situations. d. Understand that absolute value represents a number's distance from zero on the number line and use the absolute value of a rational number to represent real-world situations. e. Recognize the difference between comparing absolute values and ordering rational numbers. For	Students use inequalities to express the relationship between two rational numbers, understanding that the value of numbers is smaller moving to the left on a number line. **<u>Clarifying Notes:</u>** • Limit inequalities to simple statements of comparison (e.g., comparing a loss of 5 yards to a loss of 3 yards). • Limit comparing values with =, ≠. Recognize absolute value is a distance and not the opposite of the number. • Understand that distance is always positive (it is the direction that changes). Common models to represent and compare integers include number line models, temperature models and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers. **Operations with integers are not the expectation at this level.** In working with number line models, students internalize the order of the numbers; larger numbers on the right (horizontal) or top (vertical) of the number line and smaller numbers to the left (horizontal) or bottom (vertical) of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between two numbers.

negative rational numbers, understand that as the absolute value increases, the value of the negative number decreases.	Case 1: Two positive numbers  5 > 3 5 is greater than 3 3 is less than 5 Case 2: One positive and one negative number  3 > -3 positive 3 is greater than negative 3 negative 3 is less than positive 3 Case 3: Two negative numbers  -3 > -5 negative 3 is greater than negative 5 negative 5 is less than negative 3 <u>Example 1:</u> Write a statement to compare – 4 ½ and –2. Explain your answer. *Solution:* – 4 ½ < –2 because – 4 ½ is located to the left of –2 on the number line Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order. **6.NS.7c** Students write statements using < or > to compare rational number in context. However, explanations should reference the context rather than "less than" or "greater than".

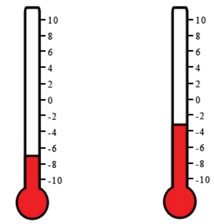
Example 1:

The balance in Sue's checkbook was –\$12.55. The balance in John's checkbook was –\$10.45. Write an inequality to show the relationship between these amounts. Who owes more?

Solution: –12.55 < –10.45, Sue owes more than John. The interpretation could also be "John owes less than Sue".

Example 2:

One of the thermometers shows -3°C and the other shows -7°C.
Which thermometer shows which temperature?
Which is the colder temperature? How much colder?
Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.



Solution:

- The thermometer on the left is -7; right is -3
- The left thermometer is colder by 4 degrees
- Either -7 < -3 or -3 > -7

Although **6.NS.7a** is limited to two numbers, this part of the standard expands the ordering of rational numbers to more than two numbers in context.

Example 3:

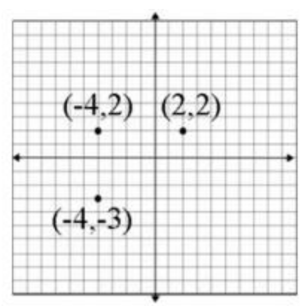
A meteorologist recorded temperatures in four cities around the world. List these cities in order from coldest temperature to warmest temperature:

Albany	5°
Anchorage	-6°
Buffalo	-7°
Juneau	-9°
Reno	12°

Solution:

Juneau	-9°
Buffalo	-7°
Anchorage	-6°
Albany	5°
Reno	12°

	6.NS.7d Students understand absolute value as the distance from zero and recognize the symbols | | as representing absolute value. Example 1: Which numbers have an absolute value of 7? Solution: 7 and –7 since both numbers have a distance of 7 units from 0 on the number line. Example 2: What is the $\lvert -3\frac{1}{2} \rvert$? Solution: $3\frac{1}{2}$ In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of 900 feet, write $\lvert -900 \rvert = 900$ to describe the distance below sea level. **6.NS.7e** When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases. For example, –24 is less than –14 because –24 is located to the left of -14 on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of –24 is greater than the absolute value of –14. For negative numbers, as the absolute value increases, the value of the negative number decreases. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars. Also understand that $-30<-20$, but a debt of 30 is greater than a debt of 20 or $\lvert -30 \rvert > \lvert -20 \rvert$.
6.NS.8 Extend knowledge of the coordinate plane to solve real-world and mathematical problems involving rational numbers. a. Plot points in all four quadrants to represent the problem.	Students find the distance between points when ordered pairs have the same x-coordinate (vertical) or same y-coordinate (horizontal). In 5th grade, students explore graphing ordered pairs in the first quadrant of the coordinate plane. This standard extends that knowledge to include all four quadrants as a result of the introduction of integers. **Clarifying Notes:** • Recognize the x-axis as a horizontal number line and the y-axis as a vertical number line.

b. Find the distance between two points when ordered pairs have the same *x*-coordinates or same *y*-coordinates. c. Relate finding the distance between two points in a coordinate plane to absolute value using a number line.	• Plot points that involve all rational numbers. Limit distance between points to horizontal distances (having the same x-coordinates) or vertical distances (having the same y-coordinates). • Recognize the final value of a distance between two points results in a positive value. <u>Example 1:</u> What is the distance between (–5, 2) and (–9, 2)? Solution: The distance would be 4 units. This would be a horizontal line since the *y*-coordinates are the same. In this scenario, both coordinates are in the same quadrant. The distance can be found by using a number line to find the distance between –5 and –9. Students could also recognize that –5 is 5 units from 0 (absolute value) and that –9 is 9 units from 0 (absolute value). Since both of these are in the same quadrant, the distance can be found by finding the difference between the distances 9 and 5. | 9 | - | 5 | = 4. Coordinates could also be in two quadrants and include rational numbers. <u>Example 2:</u> What is the distance between (-4, 2) and (2,2)? Solution: To determine the distance along the x-axis between the point (-4, 2) and (2, 2), a student must recognize that -4 is | -4 | or 4 units to the left of 0 and 2 is | 2 | or 2 units to the right of zero, so the two points are a total of 6 units apart. 

	Example 3: What is the distance between (3, $-5\frac{1}{2}$) and (3, $2\frac{1}{4}$)? Solution: The distance between (3, $-5\frac{1}{2}$) and (3, $2\frac{1}{4}$) would be $7\frac{3}{4}$ units. This would be a vertical line since the *x*-coordinates are the same. The distance can be found by using a number line to count from $-5\frac{1}{2}$ to $2\frac{1}{4}$ or by recognizing that the distance (absolute value) from $-5\frac{1}{2}$ to 0 is $5\frac{1}{2}$ units and the distance (absolute value) from 0 to $2\frac{1}{4}$ is $2\frac{1}{4}$ units so the total distance would be $5\frac{1}{2} + 2\frac{1}{4}$ or $7\frac{3}{4}$ units.
6.NS.9 Investigate and translate among multiple representations of rational numbers (fractions, decimal numbers, percentages). Fractions should be limited to those with denominators of 2, 3, 4, 5, 8, 10, and 100.	The phrase *translate among* is associated with multiple representations of a concept and indicates that given representations *a* and *b*, students must be able to convert from *a* to *b* and vice versa. If students are advanced, the teacher can use other denominators. **<u>Clarifying Notes:</u>** • Recognize ⅛ as half of ¼ to assist with conversions within all representations. • Understand that fractions with a denominator of 3 will generate a repeating decimal (limit repeating decimals to fractions with a denominator of 3). Example: Give the decimal number and percentage for the fraction $\frac{3}{4}$. Solution: $\frac{3}{4}$ can be written as the decimal 0.75 and written as the percentage 75%.

Ratios and Proportional Reasoning 6.RP

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision are: rate, ratio, unit rate, and dimensional analysis (single step).

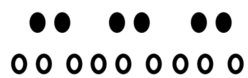
SCCCR Mathematics Standard	Unpacking What do these standards mean a child will know and be able to do?
6.RP.1 Interpret the concept of a ratio as the relationship between two quantities, including part to part and part to whole.	A ratio is the comparison of two quantities or measures. The comparison can be part to whole (ratio of guppies to all fish in an aquarium) or part to part (ratio of guppies to goldfish). Ratios can be represented symbolically and pictorially.

Clarifying Notes:

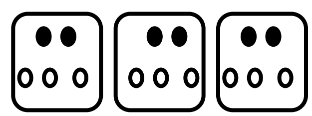
- Ratios compare two quantities.
- Simplify ratios to simplest form.

Example 1:

A comparison of 6 guppies and 9 goldfish could be expressed in any of the following forms: $\frac{6}{9}$, 6 to 9 *or* 6:9. If the number of guppies is represented by black circles and the number of goldfish is represented by white circles, this ratio could be modeled as



These values can be regrouped into 2 black circles (guppies) to 3 white circles (goldfish) which would simplify the ratio to $\frac{2}{3}$, 2 to 3, or 2:3.



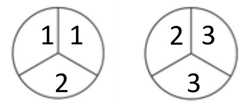
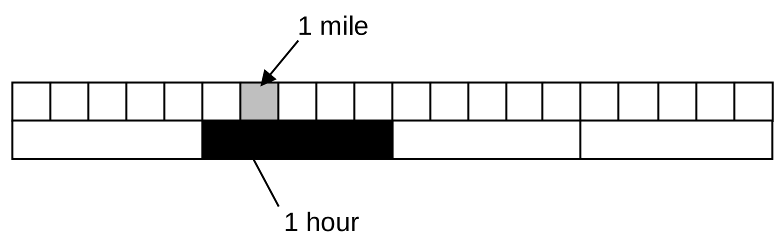
Students should be able to identify and describe any ratio using "For every _____, there are _____." In the example above, the ratio could be expressed saying, "For every 2 guppies, there are 3 goldfish."

NOTE: Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning. For example, ratios are often used to make "part-part" comparisons but fractions are not.

Part-to-part ratios are used to compare two parts. For example, the number of girls in the class (12) compared to the number of boys in the class (16) is the ratio the ratio 12 to 16.

Rates, a relationship between two units of measure, can be written as ratios, such as miles per hour, ounces per gallon and students per bus. For example, 3 cans of pudding cost $2.48 at Store A and 6 cans of the same

	pudding costs $4.50 at Store B. Which store has the better buy on these cans of pudding? Various strategies could be used to solve this problem: • A student can determine the unit cost of 1 can of pudding at each store and compare. • A student can determine the cost of 6 cans of pudding at Store A by doubling $2.48. • A student can determine the cost of 3 cans of pudding at Store B by taking ½ of $4.50.
6.RP.2 Investigate relationships between ratios and rates. a. Translate between multiple representations of ratios (i.e., a/b, a:b, a to b, visual models). b. Recognize that a rate is a type of ratio involving two different units. c. Convert from rates to unit rates.	A unit rate expresses a ratio as part-to-one, comparing a quantity in terms of one unit of another quantity. Common unit rates are cost per item or distance per time (e.g., miles per hour). Students are able to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates (e.g., miles per hour and hours per mile) are reciprocals. At this level, students should use reasoning to find these unit rates instead of an algorithm or rule. <u>Clarifying Notes:</u> • When writing a ratio, order of terminology matters. • Transfer between multiple representations of ratios. • Understand that operations with ratios are generally performed when the ratio is written in fractional form. • Students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the <u>original ratio</u> will be whole numbers. However, when creating a unit fraction, there may be a complex fraction that results with the numerator being a fraction and the denominator equaling 1. Students can • translate between the four different representations of ratios (i.e., a/b, a:b, a to b, visual models). • recognize a rate is a type of ratio using two different units. • convert from rates to unit rates and vice versa. • explain a unit rate. • define the term unit rate and demonstrate an understanding by giving several examples. • explain the difference between a ratio and a rate. A rate is a ratio where two measurements are related to each other. When discussing measurement of different units, the word rate is used rather than ratio. A unit rate expresses a ratio as part-to-one or one unit of another quantity. Students understand the unit rate from various contextual situations. For example, if a car travels 240 miles in 4 hours, then the car travels 60 miles per hour (60:1) which is a unit rate.

	<u>Example 1:</u> There are 2 cookies for 3 students. What is the amount of a cookie that each student would receive? Solution: This can be modeled by showing that there is $\frac{2}{3}$ of a cookie for 1 student, so the unit rate is $\frac{2}{3}$: 1.  <u>Example 2:</u> On a bicycle Jack can travel 20 miles in 4 hours. What are the unit rates in this situation - the distance Jack can travel in 1 hour and the amount of time required to travel 1 mile? Solution: Jack can travel 5 miles in 1 hour written as $\frac{5\,mi}{1\,hr}$ and it takes $\frac{1}{5}$ of an hour to travel each mile written as $\frac{\frac{1}{5}\text{hr}}{1\text{mi}}$. Students can represent the relationship between 20 miles and 4 hours. 
6.RP.3 Apply the concepts of ratios and rates to solve real-world and mathematical problems. a. Create a table consisting of equivalent ratios and plot the results on the coordinate plane. b. Use multiple representations, including tape diagrams, tables, double number lines, and equations, to find missing values of equivalent ratios.	Ratios and rates can be used in ratio tables and graphs to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. Scaling up or down with multiplication maintains the equivalence. To aid in the development of proportional reasoning the cross-product algorithm is ***not*** expected at this level. When working with ratio tables and graphs, ***whole number*** measurements are the expectation for this standard. <u>Clarifying Notes:</u> • Ratios can be used to find missing values in a table. • Percent is a rate per 100. • Ratio reasoning can be used to convert measurement units. • Include single step dimensional analysis (e.g., converting miles to yards).

c. Use two tables to compare related ratios. d. Apply concepts of unit rate to solve problems, including unit pricing and constant speed. e. Understand that a percentage is a rate per 100 and use this to solve problems involving wholes, parts, and percentages. f. Solve one-step problems involving ratios and unit rates (e.g., dimensional analysis).	Example 1: (see below)

Example 1:
Part A:
At Books Unlimited, 3 paperback books cost $18. What would 7 books cost? How many books could be purchased with $54?

Solution:
To find the price of 1 book, divide $18 by 3. One book costs $6.

To find the price of 7 books, multiply $6 (the cost of one book) times 7 to get $42.

To find the number of books that can be purchased with $54, create two equivalent ratios of the number of books to the cost. So, 1 book to the cost of $6 would be equivalent to an unknown number of books to the cost of $54. By multiplying $6 (the cost of one book) times 9, you would get $54. So, you need to multiply 1 book times 9 to get the solution to the number of unknown books which is 9.

Notice in the table below, a multiplicative relationship exists between the numbers.

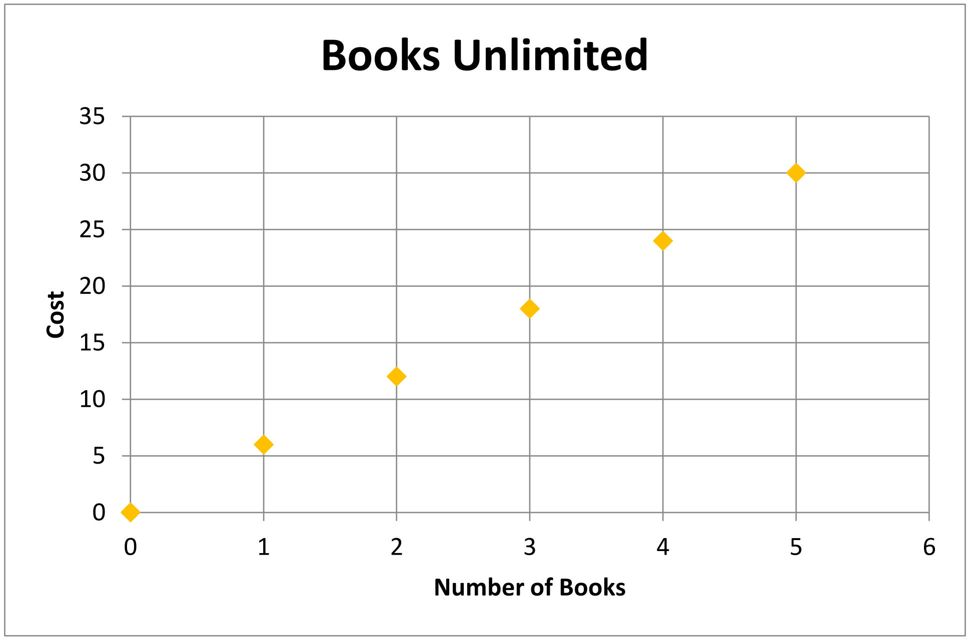
Number of Books (n)	**Cost (C)**
1	6
3	18
7	42
9	54

Part B:
Complete the table below for Books Unlimited. Then, plot the ratios as ordered pairs on a coordinate plane.

Number of Books (n)	**Cost (C)**
1	6
2	
3	18
4	
5	

Solution:

Number of Books (n)	**Cost (C)**
1	6
2	12
3	18
4	24
5	30



Students can also use tables to compare ratios.

Example 2:
Books Unlimited bookstore offers paperback books at $6 per book. Books Now bookstore offers paperback books at the prices below. What is the cost of a paperback book at Books Now? Which bookstore has the best buy? Explain your answer.

Number of Books (n)	**Cost (C)**
4	20
8	40

Solution:
Books Now offers paperback books at $5 per book based on the table above. So, Books Now offers the best buy.

Ratios can also be used in problem solving by thinking about the total amount for each ratio unit.

Example 3:
The ratio of cups of orange juice concentrate to cups of water in punch is 1:3. If James made 32 cups of punch, how many cups of orange juice did he need?

Solution:
Students recognize that the total ratio would produce 4 cups of punch. To get 32 cups, the ratio would need to be duplicated 8 times, resulting in 8 cups of orange juice concentrate.

Example 4:
Compare the number of black circles to white circles. If the ratio remains the same, how many black circles will there be if there are 60 white circles?

● ● ● ● ○ ○ ○

Black	4	40	20	60	?
White	3	30	15	45	60

Solution:
There are several strategies that students could use to determine the solution to this problem

- Add quantities from the table to total 60 white circles (15 + 45). Use the corresponding numbers to determine the number of black circles (20 + 60) to get 80 black circles.
- Use multiplication to find 60 white circles (one possibility 30 x 2). Use the corresponding numbers and operations to determine the number of black circles (40 x 2) to get 80 black circles.

Example 5:
In trail mix, the ratio of cups of peanuts to cups of chocolate candies is 3 to 2. How many cups of chocolate candies would be needed for 9 cups of peanuts?

Peanuts	Chocolate
3	2

Solution:
One possible solution is for students to find the number of cups of chocolate candies for 1 cup of peanuts by dividing both sides of the table by 3, giving $\frac{2}{3}$ cup of chocolate for each cup of peanuts. To find the amount of chocolate needed for 9 cups of peanuts, students multiply the unit rate by nine ($9 \bullet \frac{2}{3}$), giving 6 cups of chocolate.

Example 6:
If steak costs $2.25 per pound, how much does 0.8 pounds of steak cost? Explain how you determined your answer.

Solution:
The unit rate is $2.25 per pound so multiply $2.25 x 0.8 to get $1.80 per 0.8 lb of steak.

Grade 6 is a student's first introduction to percents. Percentages are a rate per 100. Models, such as percent bars or 10 x 10 grids, should be used to model percents.

	Example 7:

Example 7:

What percent is 12 out of 25?

Solution:

One possible solution method is to set up a ratio table:

Part	**Whole**
12	25
?	100

Multiply 25 by 4 to get 100. Multiplying 12 by 4 will give 48, meaning that 12 out of 25 is equivalent to 48 out of 100 or 48%.

Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (the percent).

Example 8:

What is 40% of 30?

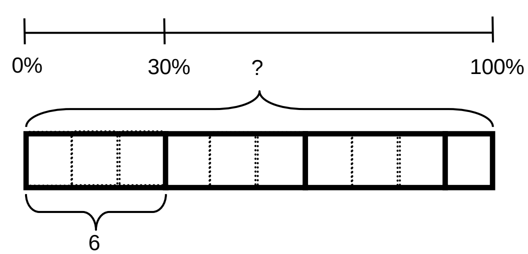
Solution:

There are several methods to solve this problem. One possible solution using rates is to use a 10 x 10 grid to represent the whole amount (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks, or 40 x 0.3, which equals 12.

Students also determine the whole amount, given a part and the percent.

Example 9:

If 30% of the students in Mrs. Rutherford's class like chocolate ice cream, then how many students are in Mrs. Rutherford's class if 6 like chocolate ice cream?



Solution:

20

Example 10:
A credit card company charges 17% interest fee on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If the bill totals $450 for this month, how much interest would you have to be paid on the balance?

Charges	$1	$50	$100	$200	$450
Interest	$0.17	$8.50	$17	$34	?

Solution:
One possible solution is to multiply 1 by 450 to get 450 and then multiply 0.17 by 450 to get $76.50.

Students use ratios as conversion factors and the identity property for multiplication to convert ratio units. A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter and centimeters per inch. Students recognize that a conversion factor is a fraction equal to 1 since the numerator and denominator describe the same quantity.

For example, $\frac{12\ inches}{1\ foot}$ is a conversion factor since the numerator and denominator equal the same amount. Since the ratio is equivalent to 1, the identity property of multiplication allows an amount to be multiplied by the ratio. Also, the value of the ratio can also be expressed as $\frac{1\ foot}{12\ inches}$.

Example 11:
How many centimeters are in 7 feet, given that 1 inch ≈ 2.54 cm?

Solution:

$$7\ feet \times \frac{12\ inches}{1\ foot} \times \frac{2.54\ cm}{1\ inch} = 7\ \cancel{feet} \times \frac{12\ \cancel{inches}}{1\ \cancel{foot}} \times \frac{2.54\ cm}{1\ \cancel{inch}} = 7\ \times 12\ \times 2.54\ cm\ \times = 213.36\ cm$$

Expressions, Equations, and Inequalities 6.EEI

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision are: base, coefficient, constant, defining the variable, equivalent expressions, exponent, grouping symbols, like terms, substitute, and term.

SCCCR Mathematics Standard	Unpacking What do these standards mean a child will know and be able to do?
6.EEI.1 Write and evaluate numerical expressions involving whole-number exponents and positive rational number bases using the Order of Operations.	Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents. The base can be a whole number, positive decimal or a positive fraction (e.g., $(\frac{1}{2})^5$ can be written $\frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2}$ which has the same value as $\frac{1}{32}$). Students recognize that an expression with a variable represents the same mathematics (e.g., x^5 can be written as $x \bullet x \bullet x \bullet x \bullet x$) and write algebraic expressions from verbal expressions. **<u>Clarifying Notes:</u>** • Perform arithmetic operations, including those involving whole-number exponents, using order of operations, including expressions with and without parentheses. The concept of order of operations is introduced in 5th grade including the use of grouping symbols, (), {}, and [ ]. <u>Example1:</u> What is the value of each of the following? a. 0.2^3 Solution: 0.008 b. $5 + 2^4 \bullet 6$ Solution: 101 c. $7^2 - 24 \div 3 + 26$ Solution: 67 <u>Example2:</u> What is the area of a square with a side length of $3x$? *Solution:* $3x \bullet 3x = 9x^2$ <u>Example 3:</u> $4^x = 64$ *Solution:* $x = 3$ because $4 \bullet 4 \bullet 4 = 64$

6.EEI.2 Extend the concepts of numerical expressions to algebraic expressions involving positive rational numbers. a. Translate between algebraic expressions and verbal phrases that include variables. b. Investigate and identify parts of algebraic expressions using mathematical terminology, including term, coefficient, constant, and factor. c. Evaluate real-world and algebraic expressions for specific values using the Order of Operations. Grouping symbols should be limited to parentheses, braces, and brackets. Exponents should be limited to whole-numbers.	**6.EEI.2a** Students write expressions from verbal descriptions using letters and numbers, understanding order is important in writing subtraction and division problems. Students understand that the expression "5 times any number, *n*" could be represented with 5*n* and that a number and letter written together means to multiply. All positive rational numbers may be used in writing expressions when operations are not expected. Students use appropriate mathematical language to write verbal expressions from algebraic expressions. It is ***important*** for students to read algebraic expressions in a manner that reinforces that the variable represents a number. **Clarifying Notes:** • Read, write, and evaluate expressions in which letters (variables) stand for numbers. • Distinguish the difference between an algebraic and numerical expression. • Identify parts of an expression using mathematical terms. • View one or more parts of an expression as a single entity. • Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Example Set 1: Students read algebraic expressions: • r + 21 as "some number plus 21" as well as "r plus 21" • n • 6 as "some number times 6" as well as "n times 6" • $^{s}/_{6}$ and s ÷ 6 as "some number divided by 6" as well as "s divided by 6" Example Set 2: Students write algebraic expressions: • 7 less than 3 times a number *Solution:* $3x - 7$ • 3 times the sum of a number and 5 *Solution:* $3(x + 5)$ • 7 less than the product of 2 and a number *Solution:* $2x - 7$ • Twice the difference between a number and 5 *Solution:* $2(z - 5)$ • The quotient of the sum of *x* plus 4 and 2 *Solution:* $\frac{x+4}{2}$ **6.EEI.2b** Students can describe expressions such as 3 (2 + 6) as the product of two factors: 3 and (2 + 6). The quantity (2 + 6) is viewed as one factor consisting of two terms. Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When

	the term is a product of a number and a variable, the number is called the coefficient of the variable. Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Variables are letters that represent numbers. There are various possibilities for the number they can represent. Consider the following expression: $x^2 + 5y + 3x + 6$ The variables are *x* and *y*. There are 4 terms which are x^2, 5y, 3x, and 6. There are 3 variable terms which are x^2, 5y, 3x. They have coefficients of 1, 5, and 3 respectively. The coefficient of x^2 is 1, since $x^2 = 1x^2$. The term 5y represent 5*y*'s or 5 • *y*. There is one constant term, 6. The expression represents a sum of all four terms. **6.EEI.2c** Students evaluate algebraic expressions, using order of operations as needed. Problems such as example 1 below require students to understand that multiplication is understood when numbers and variables are written together and to use the order of operations to evaluate. Order of operations is introduced throughout elementary grades, including the use of grouping symbols, ( ), { }, and [ ] in 5th grade. Order of operations with exponents is the focus in 6th grade. <u>Example 1:</u> Evaluate the expression 3*x* + 2*y* when *x* is equal to 4 and *y* is equal to2.4. *<u>Solution:</u>* 3 • 4 + 2 •2.4 12 + 4.8 16.8 <u>Example 2:</u> Evaluate 5(*n* + 3) – 7*n*, when *n* = ½. *Solution:* 5(½ + 3) – 7(½) 5 (3½) – 3½ Note: 7(½) = $^7/_2$ = 3½ 17½-3½ 14

	Students may also reason that 5 groups of 3½ take away 1 group of 3½ would give 4 groups of 3½. Multiply 4 times 3½ to get 14. <u>Example 3:</u> Evaluate 7*xy* when *x* = 2.5 and *y* = 9 *Solution*: Students recognize that two or more terms written together indicates multiplication. 7 (2.5)(9) = 157.5 In 5th grade students worked with the grouping symbols ( ), [ ], and { }. Students understand that the fraction bar can also serve as a grouping symbol (treats numerator operations as one group and denominator operations as another group) as well as a division symbol. <u>Example 4:</u> Evaluate the following expression when *x* = 4 and *y* =2 $\frac{x^2 + y^3}{3}$ *Solution:* $\frac{(4)^2 + (2)^3}{3}$ *substitute the values for x and y* $\frac{16+8}{3}$ *raise the numbers to the powers* $\frac{24}{3}$ *divide 24 by 3* 8 Given a context and the formula arising from the context, students could write an expression and then evaluate for any number. <u>Example 5:</u>

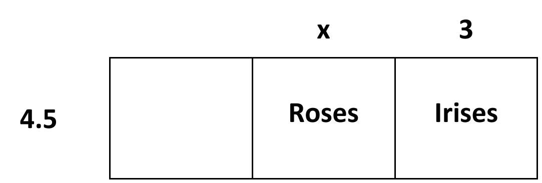
<table>
<tr>
<td></td>
<td>It costs $100 to rent the skating rink plus $5 per person. Write an expression to find the cost for any number (n) of people. What is the cost for 25people?

Solution:
The cost for any number (n) of people could be found by the expression, 100 + 5n. To find the cost of 25 people substitute 25 in for n and solve to get 100 + 5 * 25 =225.

<u>Example 6:</u>
The expression c + 0.07c can be used to find the total cost of an item with 7% sales tax, where c is the pre-tax cost of the item. Use the expression to find the total cost of an item that costs $25.
Solution: Substitute 25 in for c and use order of operations to simplify
$c + 0.07c$
$25 + 0.07(25)$
$25 + 1.75$
26.75</td>
</tr>
<tr>
<td>6.EEI.3 Apply mathematical properties (e.g., commutative, associative, distributive) to generate equivalent expressions.</td>
<td>Students use the distributive property to write equivalent expressions. Using their understanding of area models from the elementary grades, students illustrate the distributive property with variables. Properties are introduced throughout elementary grades (3.ATO.5); however, there has not been an emphasis on recognizing and naming the property. In 6th grade students are able to use the properties and identify by name as used when justifying solution methods (see example 4).

<u>Clarifying Notes:</u>
• Generate equivalent numeric and algebraic expressions.
• Understand the following mathematical properties of addition: commutative, associative, additive identity, and additive inverse
• Understand the following mathematical properties of multiplication: commutative, associative, multiplicative identity, multiplicative inverse, and distributive

<u>Example 1:</u>
Given that the width is 4.5 units and the length can be represented by x + 3, the area of the flowers below can be expressed as 4.5(x + 3) or 4.5x +13.5.



When given an expression representing area, students need to find the factors.</td>
</tr>
</table>

Example 2:

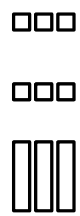
The expression 10x + 15 can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length (2x + 3). The factors (dimensions) of this figure would be 5(2x +3).

10x	**15**

Example 3:

Students use their understanding of multiplication to interpret *3 (2 + x) as 3 groups of (2 + x).* They use a model to represent x and make an array to show the meaning of *3(2 + x).* They can explain why it makes sense that *3(2 + x)* is equal to *6 +3x.*

An array with 3 columns and *x* + 2 in each column:



Students interpret *y* as referring to one *y.* Thus, they can reason that one *y* plus one *y* plus one *y* ***must be*** *3y.* They also use the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that *y* + *y* + *y* =3*y*:

Example 4:

Prove that *y* + *y* + *y* =3*y*

Solution:

y + y + y	
y • 1 + y • 1 + y • 1	Multiplicative Identity
y • (1 + 1 + 1)	Distributive Property
y • 3	
3y	Commutative Property

6.EEI.4 Apply mathematical properties (e.g., commutative, associative,	Students demonstrate an understanding of like terms as quantities being added or subtracted with the same variables and exponents. For example, 3x + 4x are like terms and can be combined as 7x;

distributive) to justify that two expressions are equivalent.	however, $3x + 4x^2$ are not like terms since the exponents with the *x* are not the same. **<u>Clarifying Notes:</u>** • Justify equivalency of numeric and algebraic expressions. This concept can be illustrated by substituting in a value for *x*. For example, $9x - 3x = 6x$ not 6. Choosing a value for *x*, such as 2, can prove non-equivalence. $9(2) - 3(2) = 6(2)$ however $9(2) - 3(2) \stackrel{?}{=} 6$ $18 - 6 = 12$ $18 - 6 \stackrel{?}{=} 6$ $12 = 12$ $12 \neq 6$ Students can also generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form. <u>Example 1:</u> Are the expressions equivalent? Explain your answer. *4m + 8* *4(m+2)* *3m + 8 + m* *2 + 2m + m + 6 + m* *Solution:* (see table below)

Expression	Simplifying the Expression	Explanation
4m + 8	*4m + 8*	Already in simplest form
4(m+2)	*4(m+2)* *4m + 8*	*Distributive property*
3m + 8 + m	*3m + 8 + m* *3m + m + 8* *4m + 8*	*Combined like terms*
2 + 2m + m + 6 + m	*2m + m + m + 2 + 6* *4m + 8*	*Combined like terms* *Combined like terms*

6.EEI.5 Understand that if any solutions exist, the solution set for an equation or inequality consists of	In elementary grades, students explored the concept of equality. In 6th grade, students explore equations as expressions being set equal to a specific value. The solution is the value of the variable that will make the equation or inequality true. Students use various processes to identify the value(s) that when

values that make the equation or inequality true.

substituted for the variable will make the equation true.

Clarifying Notes:

- Understand that with an equation you will have one solution and with an inequality you will have more than one solution.
- Verify the solution to equations or solution set to inequalities to determine if it satisfies the equation or inequality.

Example 1:
Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?

This situation can be represented by the equation $26 + n = 100$ where n is the number of papers the teacher gives to Joey. This equation can be stated as “some number was added to 26 and the result was 100.” Students ask themselves “What number was added to 26 to get 100?” to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem:

- Reasoning: 26 + 70 is 96 and 96 + 4 is 100, so the number added to 26 to get 100 is 74.
- Use knowledge of fact families to write related equations:
  $n + 26=100$, $100 – n = 26$, $100 – 26 = n$. Select the equation that helps to find n easily.
- Use knowledge of inverse operations: Since subtraction “undoes” addition then subtract 26 from 100 to get the numerical value of n.
- Scale model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balance.
- Bar Model: Each bar represents one of the values. Students use this visual representation to demonstrate that 26 and the unknown value together make 100.

100	
26	n

Solution:
Students recognize the value of 74 would make a true statement if substituted for the variable.

$26 + n = 100$
$26 + 74 =100$
$100 = 100$

Example 2:
The equation $0.44s = 11$ where s represents the number of stamps in a booklet. The booklet of

	stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies used to determine the answer. Show that the solution is correct using substitution. *Solution:* There are 25 stamps in the booklet. I got my answer by dividing 11 by 0.44 to determine how many groups of 0.44 were in 11. By substituting 25 in for *s* and then multiplying, I get 11. 0.44(25) = 11 11 = 11 Example 3: Twelve is less than 3 times another number can be shown by the inequality 12 < 3*n*. What numbers could possibly make this a true statement? *Solution:* Since 3 • 4 is equal to 12 I know the value must be greater than 4. Any value greater than 4 will make the inequality true. Possibilities are 4.13, 6, 5¾, and 200. Given a set of values, students identify the values that make the inequality true.
6.EEI.6 Write expressions using variables to represent quantities in real-world and mathematical situations. Understand the meaning of the variable in the context of the situation.	Students write expressions to represent various real-world situations. **Clarifying Notes:** • Understand that expressions do not contain equal signs. Example Set 1: • Write an expression to represent Susan's age in three years, when *a* represents her present age. • Write an expression to represent the number of wheels, *w*, on any number of bicycles. • Write an expression to represent the value of any number of quarters, *q*. *Solutions:* • a + 3 • 2*n* • 0.25*q* Given a contextual situation, students define variables and write an expression to represent the situation. Example 2: The skating rink charges $100 to reserve the place and then $5 per person. Write an expression to represent the cost for any number of people.

	n = the number of people
	
	$100 + 5n$
	
	No solving is expected with this standard; however, 6.EEI.2c does address the evaluating of the expressions.
	
	Students understand the inverse relationships that can exist between two variables. For example, if Sally has 3 times as many bracelets as Jane, then Jane has ⅓ the amount of Sally. If *m* represents the number of bracelets Sally has, then (⅓)m or $^{m}/_{3}$ represents the amount Jane has.
	
	Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.
	
	<u>Example Set 3:</u>
	
	• Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has.
	Solution: $2c + 3$ where *c* represents the number of crayons that Elizabeth has
	
	• An amusement park charges $28 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent.
	Solution: $28 + 0.35t$ where *t* represents the number of tickets purchased
	
	• Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned.
	Solution: $15h + 20 = 85$ where *h* is the number of hours worked
	
	• Describe a problem situation that can be solved using the equation $2c + 3 = 15$; where *c* represents the cost of an item.
	Possible solution:
	Sarah spent $15 at a craft store.
	    • She bought one notebook for $3.
	    • She bought 2 paintbrushes for c dollars.
	If each paint brush cost the same amount, what was the cost of one brush?
	
	• Bill earned $5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned.

	Solution: \$5.00 + *n*
6.EEI.7 Write and solve one-step linear equations in one variable involving nonnegative rational numbers for real-world and mathematical situations.	Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown, but the outcome is known. For example, in the expression, $x + 4$, any value can be substituted for the x to generate a numerical answer; however, in the equation $x + 4 = 6$, there is only one value that can be used to get a 6. Problems should be in context when possible and use only one variable. Students write equations from real-world problems and then use inverse operations to solve one-step equations based on real world situations. Equations may include non-negative fractions and decimals with non-negative solutions. Students recognize that dividing by 6 and multiplying by ⅙ produces the same result. For example, $\frac{x}{6} = 9$ and $(\frac{1}{6})x = 9$ will produce the same result. Beginning experiences in solving equations require students to understand the meaning of the equation and the solution in the context of the problem. **Clarifying Notes:** • Understand that equations contain equal signs. • Understand that the expressions on either side of the equal sign must be equivalent to one another. • Use inverse operations to solve one-step linear equations one variable involving nonnegative rational numbers (include fractions and decimals). Example 1: Meagan spent \$56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost. \$56.58 J | J | J *Sample Solution:* Students might say: "I created the bar model to show the cost of the three pairs of jeans. Each bar labeled *J* is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation $3J = \$56.58$. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than \$10 each because 10x3 is only 30 but

less than $20 each because 20x3 is 60. If I start with $15 each, I am up to $45. I have $11.58 left. I then give each pair of jeans $3. That's $9 more dollars. I only have $2.58 left. I continue until all the money is divided evenly. I ended up giving each pair of jeans another $0.86. Each pair of jeans costs $18.86(15 + 3 + 0.86). I double check that the jeans cost $18.86 each because $18.86 x 3 is $56.58."

Example 2:

Julie gets paid $20 for babysitting. She spends $1.99 on a package of trading cards and $6.50 on lunch. Write and solve an equation to show how much money Julie has left.

20		
1.99	6.50	money left over (m)

Solution: $20 = 1.99 + 6.50 + x$, $x = \$11.51$

6.EEI.8 Extend knowledge of inequalities used to compare numerical expressions to include algebraic expressions in real-world and mathematical situations.

a. Write an inequality of the form $x > c$ or $x < c$ and graph the solution set on a number line.
b. Recognize that inequalities have infinitely many solutions.

Many real-world situations are represented by inequalities. Students write inequalities to represent real world and mathematical situations. Students use the number line to represent inequalities from various contextual and mathematical situations.

Clarifying Notes:

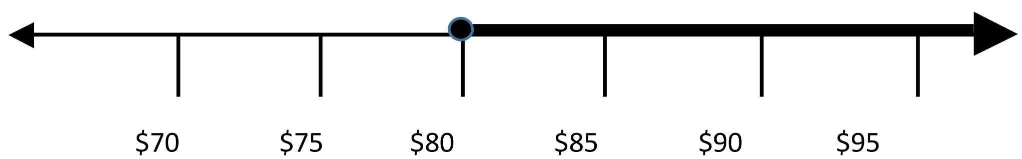
- Understand that if *x* is greater than or less than *c*, then *c* will not be in the solution set.
- Understand that solutions for *x* include rational numbers.

Example 1:

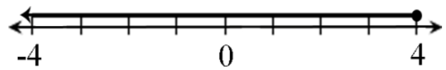
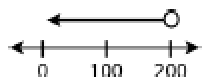
The class must raise at least $100 to go on the field trip. They have collected $20. Write an inequality to represent the amount of money, *m*, the class still needs to raise. Represent this inequality on a number line.

Solution:

The inequality $m \geq \$80$ represents this situation. Students recognize that possible values can include too many decimal values to name. Therefore, the values are represented on a number line by shading.



A number line diagram is drawn with an open circle when an inequality contains a < or > symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is

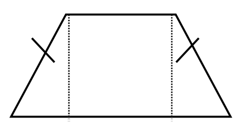
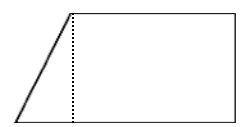
	shaded, as in the example above, when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions. <u>Example 2:</u> Graph $x \leq 4$. *Solution:*  <u>Example 3:</u> The Flores family spent less than \$200.00 last month on groceries. Write an inequality to represent this amount and graph this inequality on a number line. *Solution:* $200 > x$, where x is the amount spent on groceries. 
6.EEI.9 Investigate multiple representations of relationships in real-world and mathematical situations. a. Write an equation that models a relationship between independent and dependent variables. b. Analyze the relationship between independent and dependent variables using graphs and tables. c. Translate among graphs, tables, and equations.	The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis. Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would not be considered. A line is drawn when both variables could be represented with fractional parts. Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the x variable increases, how does the y variable change?) *Relationships should be proportional with the line passing through the origin*. Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and /or a table of values.

	Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective. **Clarifying Notes:** • Recognize that independent values represent an input (x-coordinates) and dependent values represent an output (y-coordinates). Example 1: What is the relationship between the two variables? Write an expression that illustrates the relationship. x: 1, 2, 3, 4 y: 2.5, 5, 7.5, 10 *Solution: $y = 2.5x$*

x	1	2	3	4
y	2.5	5	7.5	10

Geometric Measurement 6.GM

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision are: composing, decomposing, vertex, vertices, surface area, volume, nets, face, edge, and lateral surfaces.

SCCCR Mathematics Standard	Unpacking What do these standards mean a child will know and be able to do?
6.GM.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	Students continue to understand that area is the number of squares needed to cover a plane figure. Students should know the formulas for rectangles and triangles. "Knowing the formula" does not mean memorization of the formula. To "know" means to have an understanding of ***why*** the formula works and how the formula relates to the measure (area) and the figure. This understanding should be for *all* students. Finding the area of triangles is introduced in relationship to the area of rectangles – a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is ½ the area of the rectangle. The area of a rectangle can be found by multiplying base x height; therefore, the area of the triangle is ½bh or (b x h)/2. The following site helps students to discover the area formula of triangles. http://illuminations.nctm.org/LessonDetail.aspx?ID=L577 Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles (see figures below). Using the trapezoid's dimensions, the area of the individual triangle(s) and rectangle can be found and then added together. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites.  Isosceles trapezoid  Right trapezoid **Note:** Students recognize the marks on the isosceles trapezoid indicating the two sides have equal measure. This is the student's first exposure to the term diagonal. **<u>Clarifying Notes:</u>** • Find the total areas of trapezoids, kites, hexagons, etc. by dividing the shape into rectangles and triangles. • Derive the formula for determining the area of a triangle from the decomposition of a rectangle.

Example 1:

Find the area of a right triangle with a base length of three units, a height of four units, and a hypotenuse of 5.

Solution:

Students understand that the hypotenuse is the longest side of a right triangle. The base and height would form the 90°angle and would be used to find the area using:

$A = \frac{1}{2}bh$

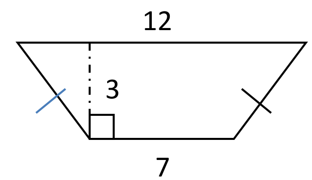
$A = \frac{1}{2}$ (3 units) (4 units)

$A = \frac{1}{2}$ 12 units^2

$A = 6$ units^2

Example 2:

Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



1st Possible Solution:

The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units^2.

The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there is a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be
½ (2.5 units) (3 units) or 3.75 units^2.

Using this information, the area of the trapezoid would be:

21 units^2
3.75 units^2
+3.75 units^2
28.5 units^2

2nd Possible Solution:

Students could put the 2 triangles together to create a rectangle that is 2.5 units by 3 units. The area of this rectangle is 7.5 units^2. Then students could find the area of the larger rectangle which is 3 units by 7 units or 21 units^2. Then students would add 21 units^2 and 7.5 units^2 to get 28.5 units^2.

Example 3:
A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?

Solution:
The new rectangle would have side lengths of 6 inches and 8 inches. The area of the original rectangle was 12 inches^2. The area of the new rectangle is 48 inches^2. The area increased 4 times (quadrupled).
Students may also create a drawing to show this visually.

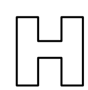
Example 4:
The lengths of the sides of a bulletin board are 4 feet by 3 feet. How many index cards measuring 4 inches by 6 inches would be needed to cover the board?

1st Possible Solution:
Change the dimensions of the bulletin board to inches (4 feet = 48 inches; 3 feet = 36 inches). The area of the board would be 48 inches x 36 inches or 1728 inches^2. The area of one index card is 24 inches^2. Divide 1728 inches^2 by 24 inches^2 to get the number of index cards. 72 index cards would be needed.

2nd Possible Solution:
48 inches ÷ 4 inches = 12 and 36 inches ÷ 6 inches = 6 so the number of cards needed is 12 x 6 which is 72 index cards.

Example 5:
The 6th -grade class at Hernandez School is building a giant wooden H for their school. The "H" will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.

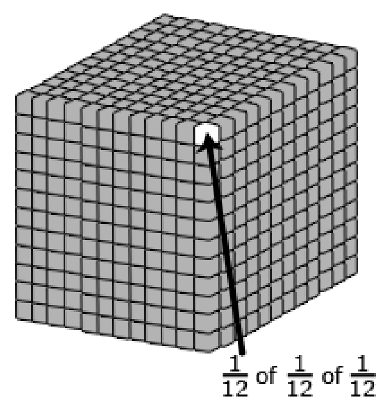
1. How large will the H be if measured in square feet?
2. The truck that will be used to bring the wood from the lumberyard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many and which dimensions) will need to be bought to complete the project?

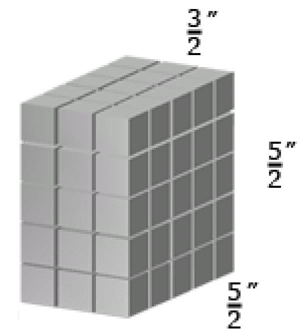


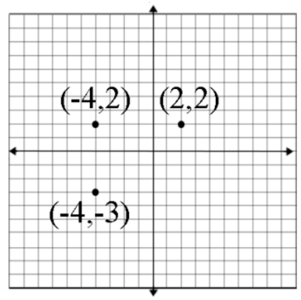
Solution:
1. One solution is to recognize that, if filled in, the area would be 10 feet tall and 10 feet wide or 100 ft^2. The size of one piece removed is 5 feet by 3.75 feet or 18.75 ft^2. There are two of these pieces.
   The area of the "H" would be 100 ft^2 – 18.75 ft^2 – 18.75 ft^2, which is 62.5ft^2.

	A second solution would be to decompose the "H" into two tall rectangles measuring 10 ft by 2.5 ft and one smaller rectangle measuring 2.5 ft by 5 ft. The area of each tall rectangle would be 25 ft^2 and the area of the smaller rectangle would be 12.5 ft^2. Therefore, the area of the "H" would be 25 ft^2 + 25 ft^2 + 12.5 ft^2or 62.5ft^2. 2. Sixty inches is equal to 5 feet, so the dimensions of each piece of wood are 5ft by 5ft. Cut **two pieces of wood in half to create four pieces 5 ft by 2.5 ft.** These pieces will make the two taller rectangles. A **third piece would be cut to measure 5ft by 2.5 ft.** to create the middle piece. <u>Example 6:</u> A border that is 2 ft wide surrounds a rectangular flowerbed 3 ft by 4 ft. What is the area of the border? *1st Possible Solution:* Two sides 4 ft. by 2 ft. would be 8ft^2 x 2 or 16 ft^2 Two sides 3 ft. by 2 ft. would be 6ft^2 x 2 or 12 ft^2 Four corners measuring 2 ft. by 2 ft. would be 4ft^2 x 4 or 16 ft^2 The total area of the border would be 16 ft^2 + 12 ft^2 + 16 ft^2 or **44 ft^2** *2nd Possible Solution:* Find the area of the entire space (the border and the flowerbed) which is 7 ft x 8 ft or 56 ft^2. Then subtract from that the area of the flowerbed which is 3 ft x 4 ft or 12 ft^2. 56 ft^2 - 12 ft^2 = **44 ft^2**.
6.GM.2 Use visual models (e.g., model by packing) to discover that the formulas for the volume of a right rectangular prism ($V = lwh, V = Bh$) are the same for whole or fractional edge lengths. Apply these formulas to solve real-world and mathematical problems.	Previously students calculated the volume of right rectangular prisms (boxes) using whole number edges. The use of models was emphasized as students worked to derive the formula V = Bh (5.MDA.3) The unit cube was 1 x 1 x 1. In 6th grade the unit cube will have fractional edge lengths. (i.e. ½ • ½ • ½). Students find the volume of the right rectangular prism with these unit cubes. Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students *derive* the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=6). In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two-dimensional shapes.

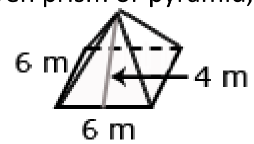
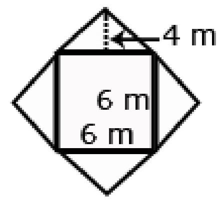
	Clarifying Notes: • Use knowledge of finding the area of a rectangle and apply that to the visual model understanding of finding the volume of right rectangular prism. • Practice with visual models using fractional side lengths so that students' interpretation of fractional side lengths multiplied together for volume is correct (e.g., ½ x ½ x ½ = ⅛). Example 1: A right rectangular prism has edges of $1\frac{1}{4}$", 1" and $1\frac{1}{2}$". How many cubes with side lengths of $\frac{1}{4}$ would be needed to fill the prism? What is the volume of the prism? *Solution:* The number of $\frac{1}{4}$" cubes can be found by recognizing the smaller cubes would be $\frac{1}{4}$" on all edges, changing the dimensions to $\frac{5}{4}$", $\frac{4}{4}$" and $\frac{6}{4}$". The number of one-fourth inch unit cubes making up the prism is 120(5 x 4 x 6). Each smaller cube has a volume of $\frac{1}{64}$ ($\frac{1}{4}$" x $\frac{1}{4}$" x $\frac{1}{4}$"), meaning 64 small cubes would make up the unit cube. Therefore, the volume is $\frac{5}{4}$ x $\frac{6}{4}$ x $\frac{4}{4}$ or $\frac{120}{64}$ (120 smaller cubes with volumes of $\frac{1}{64}$ or $1\frac{56}{64}$ → 1 unit cube with 56 smaller cubes with a volume of $\frac{1}{64}$. Example 2: The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $\frac{1}{12}$ft on each side.



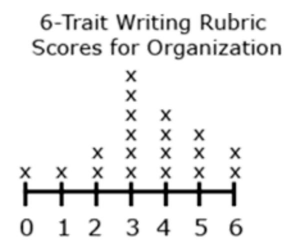
	<u>Example 3:</u> The model shows a rectangular prism with dimensions $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{5}{2}$ inches. Each of the cubic units in the model is $\frac{1}{2}$ in. on each side. Students work with the model to illustrate $\frac{3}{2}$ x $\frac{5}{2}$ x $\frac{5}{2}$ = (3 x 5 x 5) x $\frac{1}{8}$. Students reason that a small cube has volume of $\frac{1}{8}$ in^3 because 8 of them fit in a unit cube. 
6.GM.3 Apply the concepts of polygons and the coordinate plane to real-world and mathematical situations. a. Given coordinates of the vertices, draw a polygon in the coordinate plane. b. Find the length of an edge if the vertices have the same *x*-coordinates or same *y*-coordinates.	Students are given the coordinates of polygons to draw in the coordinate plane. If both *x*-coordinates are the same (2, -1) and (2, 4), then students recognize that a vertical line has been created and the distance between these coordinates is the distance between -1 and 4, or 5. If both the *y*-coordinates are the same (-5, 4) and (2, 4), then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between -5 and 2, or 7. Using this understanding, student solve real-world and mathematical problems, including finding the area and perimeter of geometric figures drawn on a coordinate plane. This standard can be taught in conjunction with **6.GM.1** to help students develop the formula for the triangle by using the squares of the coordinate grid. Given a triangle, students can make the corresponding square or rectangle and realize the triangle is ½. Students progress from counting the squares to making a rectangle and recognizing the triangle as ½ to the development of the formula for the area of a triangle. **<u>Clarifying Notes:</u>** • Recognize that edge lengths are vertical or horizontal distances in the coordinate plane. • Understand that when finding the edge length on a coordinate plane the length of the edge is always counting the spaces from point *a* to point *b* instead of counting the dots. • Understand that the edge length on a coordinate plane is a positive value regardless of the direction.

	<u>Example 1:</u> If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle? Find the area and the perimeter of the rectangle.  *Solution:* To determine the distance along the *x*-axis between the point (-4, 2) and (2, 2) a student must recognize that -4 is |-4|or 4 units to the left of 0 and 2 is |2|or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, |-4|+ |2|. The length is 6 and the width is 5. The fourth vertex would be (2, -3). The area would be 5 x 6 or 30 units^2. The perimeter would be 5 + 5 + 6 + 6 or 22 units. <u>Example 2:</u> On a map, the library is located at (-2, 2), the city hall building is located at (0,2), and the high school is located at (0,0). Represent the locations as points on a coordinate grid with a unit of 1 mile. 1. What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know? 2. What shape does connecting the three locations form? The city council is planning to place a city park in this area. How large is the area of the planned park? *Solution:* 1. The distance from the library to city hall is 2 miles. The coordinates of these buildings have the same *y*-coordinate. The distance between the *x*-coordinates is 2 (from -2 to 0). The distance from the city hall building to the high school is 2 miles. The coordinates of these buildings have the same x-coordinate. The distance between the y-coordinates is 2 (from 2 to 0). 2. The three locations form a right triangle. The area is 2 mi^2.

6.GM.4 Unfold three-dimensional figures into two-dimensional rectangles and triangles (nets) to find the surface area and to solve real-world and mathematical problems.	A net is a two-dimensional representation of a three-dimensional figure. Students represent three-dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add these sums together to find the surface area of the figure. Students construct models and nets of three-dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area. Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=205). Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure. **Clarifying Notes:** • Discover surface area of three-dimensional shapes by calculating the total area of all the faces that make up the net. Students are not using surface area formulas. • Recognize surface area as the area of the lateral surfaces and bases of a figure. • Recognize that the lateral surfaces of a figure are the faces around the figure (sides). • Understand that the figures used should be limited to rectangular and triangular prisms. Example 1: Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not?

	<u>Example 2:</u> Create the net for a given prism or pyramid, and then use the net to calculate the surface area.   *Solution:* 1. The area of the base which is 6 x 6 which is 36 m^2. 2. The area of each triangle is ½ x 6 x 4 which is 12 m^2 (there are a total of 4 triangles). 3. The surface area is 4 x 12 m^2 + 36 m^2 or 48 m^2 + 36 m^2 which is **84 m^2**.

Data Analysis and Statistics	**6.DS**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision are: statistical, non-statistical, measures of center, mean, median, mode, spread, range, interquartile range, quartile (upper and lower), mean absolute value, mean absolute deviation, shape, symmetrical (normal distribution), skewed left (negative skew), skewed right (positive skew), dot plot/line plot, histogram, box plot/box and whisker plot, sample size, measures of variability, data set, distribution, maximum value, and minimum value.	
SCCCR Mathematics Standard	Unpacking What do these standards mean a child will know and be able to do?
6.DS.1 Differentiate between statistical and non-statistical questions.	Statistics are numerical data relating to a group of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents). Students differentiate between statistical questions and those that are not. A statistical question is one that collects information that addresses differences in a population. The question is framed so that the responses will allow for the differences. For example, the question, “How tall am I?” is not a statistical question because there is only one response; however, the question, “How tall are the students in my class?” is a statistical question since the responses anticipates variability by providing a variety of possible anticipated responses that have numerical answers. Questions can result in a narrow or wide range of numerical values. Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: “How many hours per week on average do students at Jefferson Middle School exercise?” **<u>Clarifying Notes:</u>** • Recognize that a statistical question can be answered by collecting data. • Recognize that the answer to a statistical question is based on data. • Recognize that there will likely be a variety of answers to a statistical question as opposed to one single answer.
6.DS.2 Use center (mean, median, mode), spread (range, interquartile range, mean absolute value), and shape (symmetrical, skewed left, skewed right) to describe the distribution of a set of data collected to answer a statistical question.	The distribution is the arrangement of the values of a data set. Distribution can be described using center (median, mean, mode), and spread (range, interquartile range, mean absolute value). Data collected can be represented on graphs, which will show the shape of the distribution of the data. Students examine the distribution of a data set and discuss the center, spread and overall shape with dot plots, histograms and box plots.

	Clarifying Notes: • Recognize that the mean is affected by outliers. • Recognize that the median and mode are resistant to outliers. • Recognize range as the difference between the minimum and maximum values. • Recognize interquartile range as the difference between the lower (first) and upper (third) quartiles. • Recognize that the skew of a graph analyzes the symmetry or lack of symmetry of a set of data. • Recognize that if data is not skewed, then it is symmetric about the mean. • Recognize that being skewed left (negative skew) means the cluster or peak of data is to the right of the mean. • Recognize that being skewed right (positive skew) means the cluster or peak of data is to the left of the mean. • Students are expected to calculate the mean absolute deviation. • The mean absolute deviation (MAD) of a set of data is the average distance between each data value and the mean. • Understand that mean absolute deviation can be determined by finding the mean of the ranges between each data value and the mean of the data set (mean absolute value). • The larger the MAD, the *greater variability* there is in the data (the data is more spread out). • The larger the MAD, the *less reliable* the mean is as an indicator of the values within the set. Example 1: The dot plot shows the writing scores for a group of students on organization. Describe the data.  *Solution:* The values range from 0 – 6. There is a peak at 3. The median is 3, which means 50% of the scores are greater than or equal to 3 and 50% are less than or equal to 3. The mean is 3.47. If all students scored the same, the score would be 3.68.
6.DS.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	Data sets contain many numerical values that can be summarized by one number such as a measure of center. The measure of center gives a numerical value to represent the center of the data (i.e. midpoint of an ordered list or the balancing point). Another characteristic of a data set is the variability (or spread) of the values. Measures of variability are used to describe this characteristic. Students should recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

<table>
<tr>
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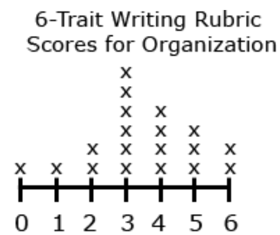
<td><u>Clarifying Notes:</u>
• Recognize that measure of center and measures of central tendencies are synonymous to one another.

<u>Example 1</u>:
Consider the data shown in the dot plot of the six trait scores for organization for a group of students.
• How many students are represented in the data set?
• What are the mean and median of the data set? What do these values mean? How do they compare?
• What is the range of the data? What does this value mean?



Solution:
• 19 students are represented in the data set.
• The mean of the data set is 3.5. The median is 3. The mean indicates that if the values were equally distributed, all students would score a 3.5. The median indicates that 50% of the students scored a 3 or higher; 50% of the students scored a 3 or lower.
• The range of the data is 6, indicating that the values vary 6 points between the lowest and highest scores.</td>
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<td>6.DS.4 Select and create an appropriate display for numerical data, including dot plots, histograms, and box plots.</td>
<td>Students display data graphically using number lines. Dot plots, histograms and box plots are three graphs to be used. Students are expected to determine the appropriate graph as well as read data from graphs generated by others. Students need to understand that this is another application of the number line.

Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.

A histogram shows the distribution of continuous data using intervals on the number line. The height of each bar represents the number of data values in that interval. In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students group the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the bin changes the appearance of the graph and the conclusions may vary from it.</td>
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A box plot is a representation of a data set that shows the five-number summary. It shows the first quartile (Q1) and the third quartile (Q3) as the left and right sides of a rectangle or a box. The median (Q2) is shown as a vertical segment inside the box. The box represents the middle half of the data. Its width is the interquartile range. The "whiskers" on the sides represent the bottom quarter and top quarter. They extend to the minimum and maximum values of the data set. A box plot can be graphed either vertically or horizontally.

Students can use applets to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations.

Box Plot Tool - http://illuminations.nctm.org/ActivityDetail.aspx?ID=77
Histogram Tool -- http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

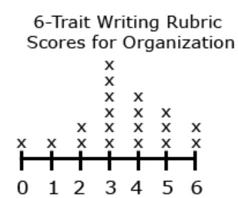
Clarifying Notes:

- Recognize that box plots and box and whisker plots are synonymous to one another.
- Understand that box plots are created and display values from the Five-Number Summary: minimum, maximum, lower quartile (first), upper quartile (third), and median.
- Understand that the lower quartile (first) is the median of the lower set of data.
- Understand that the upper quartile (third) is the median of the upper set of data.
- Understand that a box plot divides the data into fourths (quartiles).
- Recognize that the 50 percent of the data is located within the box.
- Recognize that dot plots and line plots are synonymous to one another.
- Recognize the distinct differences between bar graphs and histograms.
- Bar graphs display qualitative data and each bar represents one value.
- Histograms display qualitative and quantitative data and each bar presents a range (band) of values.

Example 1:

Nineteen students completed a writing sample that was scored on organization. The scores for organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?

Data Display Solution:



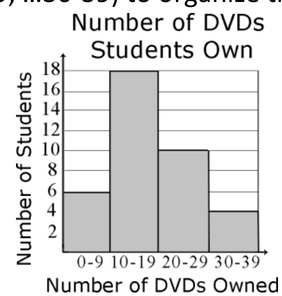
Example 2:

Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

Solution:

A histogram using 4 intervals (bins 0-9, 10-19, …30-39) to organize the data is displayed below.



Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph.

Example 3:

Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

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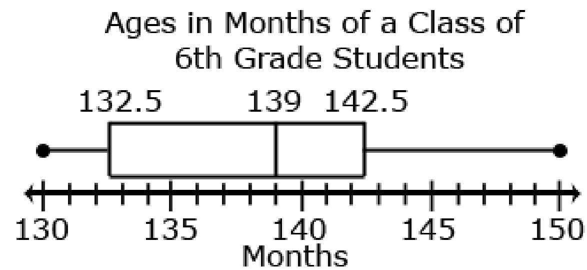
<td>Solution:
Five number summary
Minimum – 130 months
Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months
Median (Q2) – 139 months
Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months
Maximum – 150 months



This box plot shows that
¼ of the students in the class are from 130 to 132.5 months old
¼ of the students in the class are from 142.5 months to 150 months old
½ of the class are from 132.5 to 142.5 months old
The median class age is 139 months.</td>
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<tr>
<td>6.DS.5 Describe numerical data sets in relation to their real-world context.
a. State the sample size.
b. Describe the qualitative aspects of the data (e.g., how it was measured, units of measurement).
c. Give measures of center (median, mean).
d. Find measures of variability (interquartile range, mean absolute deviation) using a number line.
e. Describe the overall pattern (shape) of the distribution.
f. Justify the choices for measure of center and measure of variability based on the shape of the distribution.
g. Describe the impact that inserting or deleting a data point has on the measures of center (median, mean) for a data set.</td>
<td>Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities (addressing random sampling), the number of observations, and summary statistics. Summary statistics include quantitative measures of center (median and median) and variability (interquartile range and mean absolute deviation) including extreme values (minimum and maximum), mean, median, mode, range, and quartiles.

Students record the number of observations. Using histograms, students determine the number of values between specified intervals. Given a box plot and the total number of data values, students identify the number of data points that are represented by the box. Reporting of the number of observations must consider the attribute of the data sets, including units (when applicable).

<u>Measures of Center</u>
Given a set of data values, students summarize the measure of center with the median or mean. The median is the value in the middle of an ordered list of data. This value means that 50% of the data is greater than or equal to it and that 50% of the data is less than or equal to it.

The mean is the arithmetic average; the sum of the values in a data set divided by how many values there are in the data set. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point.

Students develop these understandings of what the mean represents by redistributing data sets to be level or fair (equal distribution) and by observing that the total distance of the data values above the mean is equal to the total distance of the data values below the mean (balancing point).</td>
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</table>

Students use the concept of mean to solve problems. Given a data set represented in a frequency table, students calculate the mean. Students find a missing value in a data set to produce a specific average.

Clarifying Notes:

- Recognize that mean absolute value and mean absolute deviation are synonymous to one another.

Example 1:

Susan has four 20-point projects for math class. Susan's scores on the first 3 projects are shown below:

Project 1: 18
Project 2: 15
Project 3: 16
Project 4: ??

What does she need to make on Project 4 so that the average for the four projects is 17? Explain your reasoning.

Solution:

One possible solution to is calculate the total number of points needed (17 x 4 or 68) to have an average of 17. She has earned 49 points on the first 3 projects, which means she needs to earn 19 points on Project 4 (68 – 49 = 19).

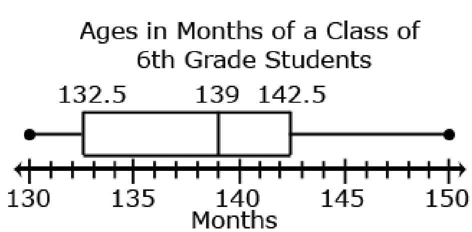
Measures of Variability

Measures of variability/variation can be described using the interquartile range or the Mean Absolute Deviation. The interquartile range (IQR) describes the variability between the middle 50% of a data set. It is found by subtracting the lower quartile from the upper quartile. It represents the length of the box in a box plot and is not affected by outliers.

Students find the IQR from a data set by finding the upper and lower quartiles and taking the difference or from reading a box plot.

Example 1:

What is the IQR of the data below:



Solution:
The first quartile is 132.5; the third quartile is 142.5. The IQR is 10 (142.5 – 132.5). This value indicates that the values of the middle 50% of the data vary by 10.

Mean Absolute Deviation (MAD) describes the variability of the data set by determining the absolute deviation (the distance) of each data piece from the mean and then finding the average of these deviations. Averaging the absolute values of the deviations leads to a measure of variation that is useful in characterizing the spread of a data distribution and in comparing distributions.

Both the interquartile range and the Mean Absolute Deviation are represented by a single value. Students recognize that with a measure of variability, higher values represent a great variability in the data while lower values represent less variability in the data.

<u>Example 2:</u>
The following data set represents the size of 9 families:
3, 2, 4, 2, 9, 8, 2, 11, 4.
What is the MAD for this data set?

Solution:
The mean is 5. The MAD is the average variability of the data set. To find the MAD:
1. Find the deviation from the mean.
2. Find the absolute deviation for each of the values from step 1
3. Find the average of these absolute deviations.

The table below shows these calculations:

Data Value	**Deviation from Mean**	**Absolute Deviation**
3	-2	2
2	-3	3
4	-1	1
2	-3	3
9	4	4
8	3	3
2	-3	3
11	6	6
4	-1	1
	MAD	26/9 = 2.89

	This value indicates that on average family size varies 2.89 from the mean of 5. Students understand how the measures of center and measures of variability are represented by graphical displays. Students describe the context of the data, using the shape of the data and are able to use this information to determine an appropriate measure of center and measure of variability. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.