South Carolina College- and Career-Ready Standards for Mathematics

Standards Unpacking Documents

Grade 5

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Standards Unpacking Documents – Grade 5

With the final approval of the *South Carolina College- and Career-Ready Standards for Mathematics* on March 11, 2015, educators were provided with clear, rigorous, and coherent standards for mathematics that would prepare students for success in their intended career paths that will either lead directly to the workforce or further education in post-secondary institutions. *South Carolina College- and Career-Ready Standards for Mathematics* contains South Carolina College- and Career-Ready (SCCCR) Content Standards for Mathematics that represent a balance of conceptual and procedural knowledge and specify the mathematics that students will master in each grade level and high school course.

The State Department of Education released Support Documents throughout the 2015-2016 school year to provide support for educators who are implementing the *South Carolina College- and Career-Ready Standards for Mathematics*. The Support Documents, which are organized by grades, are then organized by possible units of study which address all of the standards for that grade. The Support Documents can be found at http://ed.sc.gov/instruction/standards-learning/mathematics/support-documents-and-resources/. The purpose of these documents is to provide guidance as to how all the standards at each grade may be grouped into units. Since these documents are merely guidance, the State Department of Education encourages districts to implement the standards in a manner that best meets the needs of students.

To provide an additional supportive resource for South Carolina mathematics educators and continue to build upon the work of the State Department of Education, the South Carolina Leaders of Mathematics Education organization offered to create grade specific Standards Unpacking Documents. These documents would be organized by grade level and grouped by key concept. The *South Carolina College- and Career-Ready Standards for Mathematics* and the South Carolina grade specific Mathematics Support Documents as well as North Carolina and Kansas resources were utilized in the creation of the grade specific Standards Unpacking Documents. This document was adapted and modified specifically from the North Carolina Department of Education grade specific Mathematics Unpacked Content resources as well as the Kansas Association of Teachers of Mathematics Flip Books.

The Mathematics Standards Unpacking Documents were collaboratively written by South Carolina classroom teachers, instructional coaches, district leaders, and higher education faculty who are members of the South Carolina Leaders of Mathematics Education. It is with sincere appreciation that we humbly acknowledge the dedication, hard work and generosity of time provided by the members of the South Carolina Leaders of Mathematics Education who made the Mathematics Standards Unpacking Documents possible.

The primary purpose and goal of the Mathematics Standards Unpacking Documents are to assist and support educators who are teaching the *South Carolina College- and Career-Ready Standards for Mathematics* and to increase student achievement by ensuring educators understand specifically what the standards mean a student must know, understand and be able to do. These documents may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. These documents, along with on-going professional development, may be one of many resources used to understand and teach *South Carolina College- and Career-Ready Standards for Mathematics*.

South Carolina College- and Career-Ready Standards for Mathematics
Mathematical Process Standards

The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Standards for Mathematics for each grade level and course. Since the Process Standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the Content Standards, the Process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

Students who are college- and career-ready take a productive and confident approach to mathematics. They are able to recognize that mathematics is achievable, sensible, useful, doable, and worthwhile. They also perceive themselves as effective learners and practitioners of mathematics and understand that a consistent effort in learning mathematics is beneficial.

The Program for International Student Assessment defines mathematical literacy as "an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens" (Organization for Economic Cooperation and Development, 2012).

A mathematically literate student can:

1. **Make sense of problems and persevere in solving them.**
    a. Relate a problem to prior knowledge.
    b. Recognize there may be multiple entry points to a problem and more than one path to a solution.
    c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem.
    d. Evaluate the success of an approach to solve a problem and refine it if necessary.

2. **Reason both contextually and abstractly.**
    a. Make sense of quantities and their relationships in mathematical and real-world situations.
    b. Describe a given situation using multiple mathematical representations.
    c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation.
    d. Connect the meaning of mathematical operations to the context of a given situation.

3. **Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others.**
    a. Construct and justify a solution to a problem.
    b. Compare and discuss the validity of various reasoning strategies.
    c. Make conjectures and explore their validity.
    d. Reflect on and provide thoughtful responses to the reasoning of others.

4. **Connect mathematical ideas and real-world situations through modeling.**
    a. Identify relevant quantities and develop a model to describe their relationships.
    b. Interpret mathematical models in the context of the situation.
    c. Make assumptions and estimates to simplify complicated situations.
    d. Evaluate the reasonableness of a model and refine if necessary.

5. **Use a variety of mathematical tools effectively and strategically.**
    a. Select and use appropriate tools when solving a mathematical problem.
    b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.

6. **Communicate mathematically and approach mathematical situations with precision.**
    a. Express numerical answers with the degree of precision appropriate for the context of a situation.
    b. Represent numbers in an appropriate form according to the context of the situation.
    c. Use appropriate and precise mathematical language.
    d. Use appropriate units, scales, and labels.

7. **Identify and utilize structure and patterns.**
    a. Recognize complex mathematical objects as being composed of more than one simple object.
    b. Recognize mathematical repetition in order to make generalizations.
    c. Look for structures to interpret meaning and develop solution strategies.

Number Sense and Base Ten	5.NSBT

Overview

Students will build on their understanding of the place value system by discovering that a digit in one place of a multi-digit whole number represents 1/10 times what the same digit in the place to its left represents. This understanding supports student work with exponents in whole and decimal numbers. Students will extend their understanding of the base-ten system to compare and round decimal numbers to thousandths. Students perform operations with multi-digit whole numbers and with decimals to hundredths. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. Students develop fluency in addition, subtraction, multiplication, and division computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision are: **place value, decimal, decimal point, exponent, powers of ten, pattern, tenths, hundredths, thousandths, expanded form, inequality, expression, factor, product, quotient, divisor, dividend.**

SCCCR Mathematics Standard	**Unpacking** What do these standards mean a child will know and be able to do?
5.NSBT.1 Understand that, in a multi-digit whole number, a digit in one place represents 10 times what the same digit represents in the place to its right, and represents $\frac{1}{10}$ times what the same digit represents in the place to its left.	Students extend their understanding of the base-ten system to the relationship between adjacent places in whole and decimal numbers. This standard calls for students to reason about the magnitude of numbers. They use their understanding of unit fractions to compare decimal places and fractional language to describe those comparisons. Before considering the relationships in decimal numbers, students express their understanding that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and 1/10 of what it represents in the place to its left. In 4^{th} grade, students came to understand that "in a multi-digit whole number, a digit represents ten times what the same digit represents in the place to its right." Now in 5^{th} grade students review that understanding and extend it to the relationship between a digit and the digit to its left. In Grade 5, the concept of place value is extended to include decimal values to thousandths. The strategies used with place value for Grades 3 and 4 should be drawn upon and extended for whole numbers and decimal numbers. For example, students need to continue to represent, write and state the value of numbers including decimal numbers. For students who are not able to read, write and represent multi-digit numbers, working with decimals will be challenging. Money is a good medium to compare the place values of decimals. Present contextual situations that require the comparison of the cost of two items to determine the lower or higher priced item. Using such contextual

situations, students should be able to identify how many pennies, dimes, dollars and ten dollars, etc., are in a given value and those relate that to the place value relationships. Build on the understanding that it always takes ten of the number to the right to make the number to the left.

Example:
What can you share about the digit 4 in the numbers 542 and 324? The digit 2?

The 4 in 542 represents 4 tens or 40 and the 4 in 324 represents 4 ones or 4. Since the 4 in 324 is one place to the right of the 4 in 542 the value of the 4 in the number 324 is 1/10 of its value in the number 542. Meanwhile, the 2 in the number 542 is different from the value of the 2 in 324. The 2 in 542 represents 2 ones or 2, while the 2 in 324 represents 2 tens or 20. Since the 2 in 324 is one place to the left of the 2 in 542 the value of the 2 is 10 times greater.

Example:
Compare the hundreds and tens place of the number 5555.

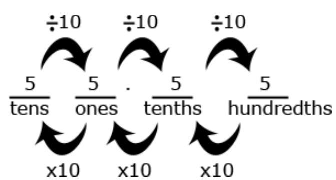
A student thinks, "I know that in the number 5555, the 5 in the tens place (5555) represents 50 and the 5 in the hundreds place (5555) represents 500. So a 5 in the hundreds place is ten times as much as a 5 in the tens place or a 5 in the tens place is 1/10 of the value of a 5 in the hundreds place."

Example:
Compare the digit 8 in the numbers 845 and 782.

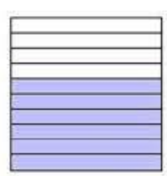
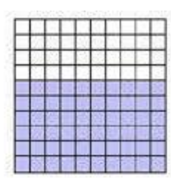
Based on the base-10 number system, the 8 in 845 has a value of 800 which is ten times as much as the 8 in the number 782. In the same spirit, the 8 in 782 is 1/10 the value of the 8 in 845.

To extend this understanding of place value to their work with decimals, students use a model of one unit. They cut it into 10 equal pieces, shade in, or describe 1/10 of that model using fractional language such as "This is 1 out of 10 equal parts. So it is 1/10. I can write this using 1/10 or 0.1." They repeat the process by finding 1/10 of a 1/10 (e.g., dividing 1/10 into 10 equal parts to arrive at 1/100 or 0.01) and can explain their reasoning as "0.01 is 1/10 of 1/10 thus is 1/100 of the whole unit." Decimal blocks and/or cards are also good resources.

In the number 55.55, each digit is 5, but the value of the digits is different because of the placement.

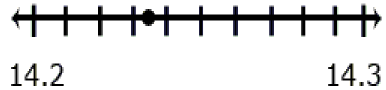


5.NSBT.2 Use whole number exponents to explain: a. patterns in the number of zeroes of the product when multiplying a number by powers of 10; b. patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.	New at Grade 5 is the use of whole number exponents to denote powers of 10. This standard includes multiplying by multiples of 10 and powers of 10, including 10^2 which is 10 x 10=100, and 10^3 which is 10 x 10 x 10=1,000. Students should have experiences working with connecting the pattern of the number of zeroes in the product when you multiply by powers of 10. Patterns in the number of 0s in products of a whole number and a power of 10 and the location of the decimal point in products of decimals with powers of 10 can be explained in terms of place value. Because students have developed their understandings of and computations with decimals in terms of multiples rather than powers, connecting the terminology of multiples with that of powers affords connections between understanding of multiplication and exponentiation. (*Progressions for the CCSSM, Number and Operation in Base Ten*, CCSS Writing Team, April 2011, page 16) <u>Example:</u> **When solving 2.5 x 10^3, explain the pattern in the placement of the decimal point.** 2.5 x 10^3 = 2.5 x (10 x 10 x 10) = 2.5 x 1,000 = 2,500 Students should reason that the exponent with the 10 indicates how many places the decimal point is moving (not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times) when you multiply by a power of 10. Since we are multiplying by a power of 10 the decimal point moves to the right. <u>Example:</u> **When solving 350 ÷ 10^3, use your understanding of the placement of the decimal point when dividing.** Possible solutions: • 350 ÷ 10^3 = 350 ÷ 1,000 = 350/1000 = 0.350 = 0.35 I know that 10^3 is equal to 1,000 and that 350 ÷ 1000 can also be written as 350/1000 which is read as 350 thousandths. I can now write 0.350 and that is the same as 0.35. • I know that 350 ÷ 10 = 35. So now I need to solve 35 ÷ 10 = 3.5 which means I have divided by 10 twice. I need to divide once more by 10, so 3.5 ÷ 10 = 0.35 These possible solutions show that when we divide by powers of 10, the exponent above the 10 indicates how many places the decimal point is moving (how many times we are dividing by 10 , the number becomes ten times smaller). Since we are dividing by powers of 10, the decimal point moves to the left. Students need to be provided with opportunities to explore this concept and come to this understanding. It should not just be taught procedurally.

	<u>Example:</u> **Investigate the patterns in the number of zeroes of the product when multiplying a number by powers of 10.** Students might write: • $36 \times 10 = 36 \times 10^1 = 360$ • $36 \times 10 \times 10 = 36 \times 10^2 = 3600$ • $36 \times 10 \times 10 \times 10 = 36 \times 10^3 = 36{,}000$ • $36 \times 10 \times 10 \times 10 \times 10 = 36 \times 10^4 = 360{,}000$ Students might think and/or say: • I noticed that every time I multiplied by 10 I added a zero to the end of the number. That makes sense because each digit's value became 10 times larger. To make a digit 10 times larger, I move the decimal one place to the right. • When I multiplied 36 by 10, the 30 became 300. The 6 became 60 or the 36 became 360. So I had to add a zero at the end to have the 3 represent 3 one-hundreds (instead of 3 tens) and the 6 represents 6 tens (instead of 6 ones). Students should be able to use the same type of reasoning as above to explain why the following multiplication and division problem by powers of 10 make sense. • $523 \times 10^3 = 523{,}000$ • $5.223 \times 10^2 = 522.3$ • $52.3 \div 10^1 = 5.23$
5.NSBT.3 Read and write decimals in standard and expanded form. Compare two decimal numbers to the thousandths using the symbols >, =, or <.	Students build on the understanding they developed in fourth grade to read, write, and compare decimals to hundredths. Expanded form is included to build upon work in 5.NBT.2 and deepen students' understanding of place value. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. They use concrete models and number lines to extend this understanding to decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc. They read decimals using fractional language and write decimals in decimal form, as well as in expanded notation. This investigation leads them to understanding equivalence of decimals (0.8 = 0.80 = 0.800).  0.6 six tenths  0.60 sixty hundredths 

	Comparing decimals builds on work from fourth grade. <u>Example:</u> **How many equivalent forms of 0.72 can you list?** Some equivalent forms of 0.72 are: 72/100 70/100 + 2/100 7/10 + 2/100 0.720 7 x (1/10) + 2 x (1/100) 7 x (1/10) + 2 x (1/100) + 0 x (1/1000) 0.70 + 0.02 720/1000 Students need to understand the size of decimal numbers and relate them to common benchmarks such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals. <u>Example:</u> **Compare 0.25 and 0.17 using the symbols >, =, or <.** A student might think, "25 hundredths is more than 17 hundredths". They may also think that it is 8 hundredths more. They may write this comparison as 0.25 > 0.17 and recognize that 0.17 < 0.25 is another way to express this comparison. <u>Example:</u> **Compare 0.207 and 0.26 using the symbols >, =, or <.** A student might think, "Both numbers have 2 tenths, so I need to compare the hundredths. The second number has 6 hundredths and the first number has no hundredths so the second number must be larger. Another student might think while writing fractions, "I know that 0.207 is 207 thousandths (and may write 207/1000). 0.26 is 26 hundredths (and may write 26/100) but I can also think of it as 260 thousandths (260/1000). So, 260 thousandths is more than 207 thousandths.
5.NSBT.4 Round decimals to any given place value within thousandths.	This standard refers to rounding. Students should go beyond simply applying an algorithm or procedure for rounding. The expectation is that students have a deep understanding of decimal place value, decimal number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding. <u>Example:</u> **While situations are more meaningful in content, an example that could be placed in context is round 14.235 to the nearest tenth.**

Students recognize that the answer must be in tenths. Using a number line and approximating the location of 14.23, they can determine that it will round to either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30).



Students should use benchmark numbers to support this work. Benchmarks are convenient numbers for comparing and rounding numbers.   0, 0.5, 1, 1.5 are examples of benchmark numbers.

5.NSBT.5 Fluently multiply multi-digit whole numbers using strategies to include a standard algorithm.

This standard refers to fluency which means accuracy (correct answer), efficiency (a reasonable amount of steps), and flexibility (using strategies such as the distributive property or breaking numbers apart). This standard builds upon students' work with multiplying numbers in third and fourth grade.

In third grade, students developed conceptual understanding of multiplication and in fourth grade, students extended that understanding to multiply a four-digit number by a one-digit number and to multiply a two-digit number by a two-digit number using such strategies as properties of operations and place value. In fifth grade, the focus is on linking those conceptual understandings to a standard algorithm for the purpose of developing fluency.

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

<u>Examples using computation strategies:</u>
Note: These examples are based on properties of operations and not on a standard algorithm as required by the standard.

The book company printed 452 books. Each book had 150 pages. How many pages did the book company print?

Strategy 1	Strategy 2
452 x 150 452 (15 x 10) 452 x 15 = 6,780 6,780 x 10 = 67,800	452 x 150 452 (100 + 50) 452 x 100 = 45,200 452 x 50 = 22,600 45,200 + 22,600 = 67,800

There are 225 dozen cookies in the bakery. How many cookies are there?

Student 1	Student 2	Student 3
225 x 12 I broke 12 up into 10 and 2. 225 x 10 = 2,250 225 x 2 = 450 2,250 + 450 = 2,700	225 x 12 I broke up 225 into 200 and 25. 200 x 12 = 2,400 I broke 25 up into 5 x 5, so I had 5 x 5 x 12 or 5 x 12 x 5. 5 x 12= 60. 60 x 5 = 300 I then added 2,400 and 300 2,400 + 300 = 2,700.	I doubled 225 and cut 12 in half to get 450 x 6. I then doubled 450 again and cut 6 in half to get 900 x 3. 900 x 3 = 2,700.

Example using a computation algorithm:
Note: This example does use an algorithm which is required by the standard.

Find the product of 123×34.
When students apply a standard algorithm, they may decompose 34 into 30+4. Then they multiply 123 by 4, the value of the number in the ones place, and then multiply 123 by 30, the value of the 3 in the tens place, and add the two products. The ways in which students are taught to record this method may vary, but ALL should emphasize the place-value nature of the algorithm. For example, one might write

```
  123
 ×34
  492        this is the product of 4 and 123
 3690        this is the product of 30 and 123
 4182        this is the product of the two partial products
```

Note that a further decomposition of 123 into 100 + 20 + 30 and recording of the partial products would also be acceptable.

<table>
<tr>
<td></td>
<td>
<u>Example:</u>
NOTE: The following example is a model which links the conceptual understanding to the algorithm but is not acceptable in and of itself to meet the standard expectation.

Draw an area model for 225 x 12.

<table>
<tr><td></td><td>200</td><td>20</td><td>5</td><td></td></tr>
<tr><td>10</td><td>200x10=
2,000</td><td>20x10=
200</td><td>5x10=
50</td><td rowspan="2">2,000
400
200
40
50
<u>+ 10</u>
2,700</td></tr>
<tr><td>2</td><td>200x2=
400</td><td>20x2=
40</td><td>5x2=
10</td></tr>
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<tr>
<td>5.NSBT.6 Divide up to a four-digit dividend by a two-digit divisor, using strategies based on place value, the properties of operations, and the relationship between multiplication and division.</td>
<td>
This standard references various strategies for division. Division problems can include remainders. This standard implies more than simply computation as the connection to story contexts is critical. Make sure students are exposed to problems where the divisor is the number of groups and where the divisor is the size of the groups.

In fourth grade, students’ experiences with division were limited to dividing by one-digit divisors. This standard extends students’ prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a “familiar” number, a student might decompose the dividend using place value.

In fifth grade, students fluently compute products of whole numbers using a standard algorithm. Underlying this algorithm are the properties of operations and the base-ten system. Division strategies in fifth grade involve breaking the dividend apart into like base-ten units and applying the distributive property to find the quotient place by place, starting from the highest place. (Division can also be viewed as finding an unknown factor: the dividend is the product, the divisor is the known factor, and the quotient is the unknown factor.)

Students continue their fourth grade work on division, extending it to computation of whole number quotients with dividends of up to four digits and two-digit divisors. Estimation becomes relevant when extending to two-digit divisors. Even if students round appropriately, the resulting estimate may need to be adjusted.
</td>
</tr>
</table>

Recording division after an underestimate

1655 ÷ 27

Rounding 27 to 30 produces the underestimate 50 at the first step but this method allows the division process to be continued

```
                 1  \
                10   ) 61
       (30)     50  /
         27 ) 1655
             -1350
               305
              -270
                35
               -27
                 8
```

(*Progressions for the CCSSM, Number and Operation in Base Ten*, CCSS Writing Team, April 2011, page 16)

<u>Example:</u>

There are 1,716 students participating in Field Day. They are put into teams of 16 for the competition. How many teams are created? If there is a remainder, how does that relate to the problem?

Student 1

1,716 divided by 16

There are 100 16's in 1,716.
1,716 – 1,600 = 116
I know there are at least 6 16's.
116 - 96 = 20
I can take out at least 1 more 16.
20 - 16 = 4

There were 107 teams with 4 students left over. If we put the extra students on different teams, 4 teams will have 17 students.

Student 2

1,716 divided by 16.

There are 100 16's in 1,716.
Ten groups of 16 is 160. That's too big.
Half of that is 80, which is 5 groups.
I know that 2 groups of 16's is 32.
I have 4 students left over. There are 107 teams with 4 students left over.
If we put the extra students on different teams, then 4 teams will have 17 students and 103 teams will have 16 students.

1716 -1600	100
116 -80	5
36 -32	2
4	

Student 3

1,716 ÷ 16 =

I want to get to 1,716.
I know that 100 16’s equals 1,600.
I know that 5 16’s equals 80.
1,600 + 80 = 1,680
Two more groups of 16’s equals 32, which gets us to 1,712.
I am 4 away from 1,716.
So we had 100 + 6 + 1 = 107 teams.
Those other 4 students can just hang out.

Student 4

How many 16’s are in 1,716?

We have an area of 1,716. I know that one side of my array is 16 units long. I used 16 as the height. I am trying to answer the question what is the width of my rectangle if the area is 1,716 and the height is 16. 100 + 7 = 107 R 4. There are 107 teams of 16 students and the 4 extra students can be helpers for Field Day.

	100	**7**
16	100 x 16 = 1,600	7 x 16 =112
	1,716 - 1,600 = 116	116 - 112 = 4

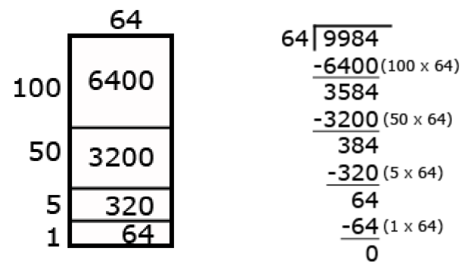
<u>Example:</u>
Using expanded notation to solve 2682 ÷ 25.

2682 ÷ 25 = (2000 + 600 + 80 + 2) ÷ 25

Using understanding of the relationship between 100 and 25, a student might think

- I know that 100 divided by 25 is 4 so 200 divided by 25 is 8 and 2000 divided by 25 is 80.
- 600 divided by 25 has to be 24.
- Since 3 x 25 is 75, I know that 80 divided by 25 is 3 with a reminder of 5. (Note that a student might divide into 82 and not 80)
- I can’t divide 2 by 25 so 2 plus the 5 leaves a remainder of 7.
- 80 + 24 + 3 = 107. So, the answer is 107 with a remainder of 7.

Using an equation that relates division to multiplication, 25 x *n* = 2682, a student might estimate the answer to be slightly larger than 100 because he recognizes that 25 x 100 = 2500.

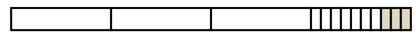
	Example: **Solve. 9984 ÷ 64** An area model for division is shown below. As the student uses the area model, he keeps track of how much of the 9984 is left to divide.  The answer is 100 + 50 + 5 + 1 = 156.
5.NSBT.7 Add, subtract, multiply, and divide decimal numbers to hundredths using concrete area models and drawings.	This standard builds on the work from fourth grade where students are introduced to decimals and compare them using visual and concrete models. Fifth grade students continue to compare decimals (5.NSBT.3) but as is noted later in this document, do so using symbols. Regarding operations, fifth grade students develop conceptual understanding of adding, subtracting, multiplying and dividing decimals to hundredths by using concrete area models and drawings. This work should focus on concrete models and pictorial representations, rather than relying on an algorithm. The use of symbolic notations involves having students record the answers to computations (2.25 x 3= 6.75), but this work should not be done without models or pictures. This standard includes students' reasoning and explanations of how they use models, pictures, and strategies. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers. Examples: **Estimate the sum of the following: 3.6 + 1.7** A student might estimate the sum to be larger than 5 because 3.6 is more than 3 ½ and 1.7 is more than 1 ½. **Estimate the difference of the following: 5.4 – 0.8** A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.

Estimate the product of the following: 6 x 2.4

A student might estimate an answer between 12 and 18 since 6 x 2 is 12 and 6 x 3 is 18. Another student might give an estimate of a little less than 15 because she/he figures the answer to be very close, but smaller than 6 x 2 ½ and thinks of 2 ½ groups of 6 as 12 (2 groups of 6) + 3 (½ of a group of 6).

Find the difference of the following using a drawing to justify your solution: 4 - 0.3

3 tenths subtracted from 4 wholes. The wholes must be divided into tenths. (solution is 3 and 7/10 or 3.7)



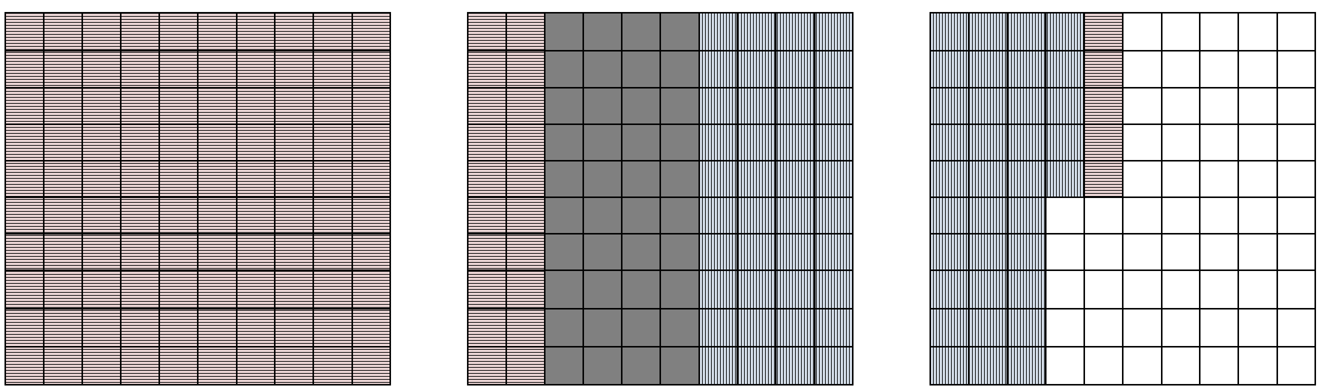
Addition Example:

A recipe for a cake requires 1.25 cups of milk, 0.40 cups of oil, and 0.75 cups of water. How much liquid is in the mixing bowl?

Student 1

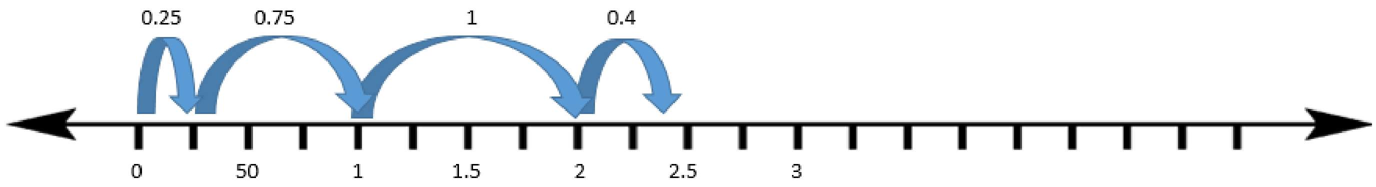
1.25 + 0.40 + 0.75

First, I broke the numbers apart:
I broke 1.25 into 1.00 + 0.20 + 0.05
I left 0.40 like it was.
I broke 0.75 into 0.70 + 0.05
I combined my two 0.05s to get 0.10
I combined 0.40 and 0.20 to get 0.60
I added the 1 whole from 1.25
I ended up with 1 whole, 6 tenths, 7 more tenths and 1 more tenth which equals 2.40 cups.



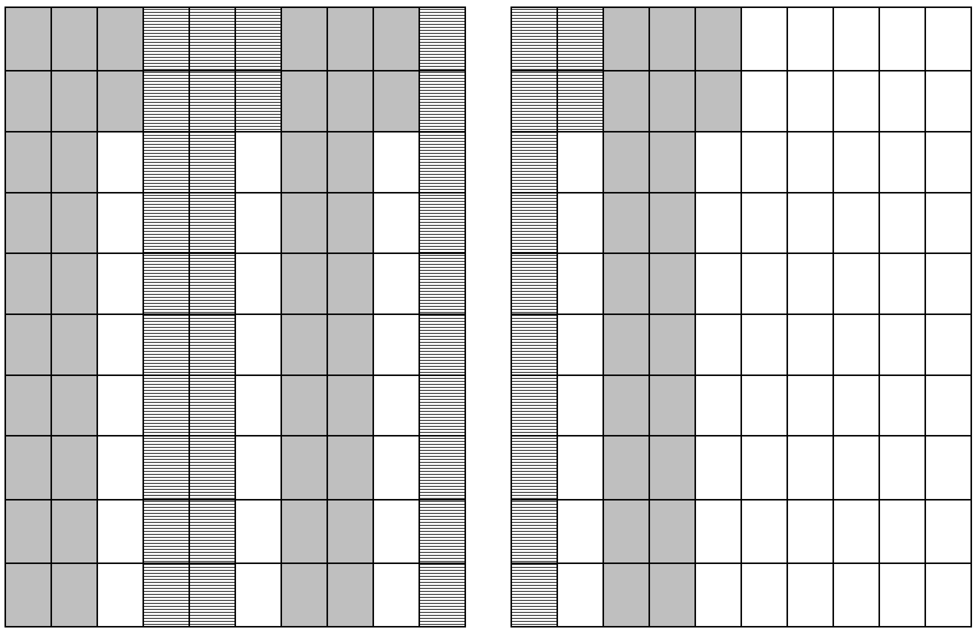
Student 2

I saw that the 0.25 in 1.25 and the 0.75 for water would combine to equal 1 whole.
I then added the 2 wholes and the 0.40 to get 2.40.



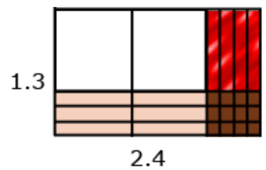
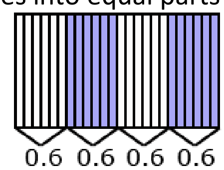
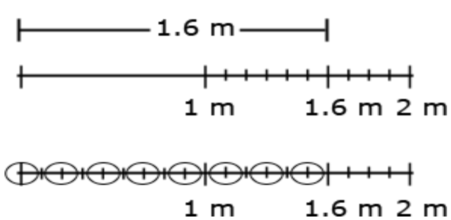
Multiplication Example Solved with Concept of Repeated Addition:

A gumball costs $0.22. How much do 5 gumballs cost? Estimate the total, and then calculate. Was your estimate close?



I estimate that the total cost will be a little more than a dollar. I know that 5 20's equal 100 and we have 5 22's. I have 10 whole columns shaded and 10 individual boxes shaded. The 10 columns equal 1 whole. The 10 individual boxes equal 10 hundredths or 1 tenth. My answer is $1.10. My estimate was a little more than a dollar, and my answer was $1.10. I was really close.

Additional multiplication and division examples:

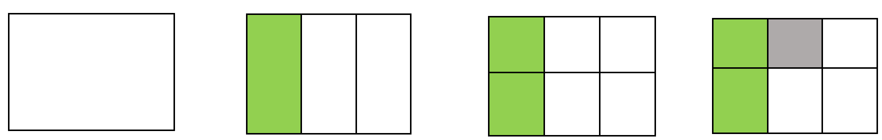
An area model can be useful for illustrating products.  $\begin{array}{r} 2.4 \\ \underline{\times\ 1.3} \\ .12 \\ .60 \\ .40 \\ \underline{+\ 2.00} \\ 3.12 \end{array}$	Students should be able to describe the partial products displayed by the area model. For example, "3/10 times 4/10 is 12/100. 3/10 times 2 is 6/10 or 60/100. 1 group of 4/10 is 4/10 or 40/100. 1 group of 2 is 2."
Example of division: finding the number in each group or share. Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as $2.4 \div 4 = 0.6$  	
Example of division: finding the number of groups. Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many can he cut? Students could draw a segment to represent 1.6 meters. In doing so, she/he would count in tenths to identify the 6 tenths, and be able identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths.  	Students might count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as 10/10, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths, . . .16 tenths, a student can count 8 groups of 2 tenths. Use their understanding of multiplication and think, "8 groups of 2 is 16, so 8 groups of 2/10 is 16/10 or 1 6/10."

Number Sense and Fractions 5.NSF

Overview

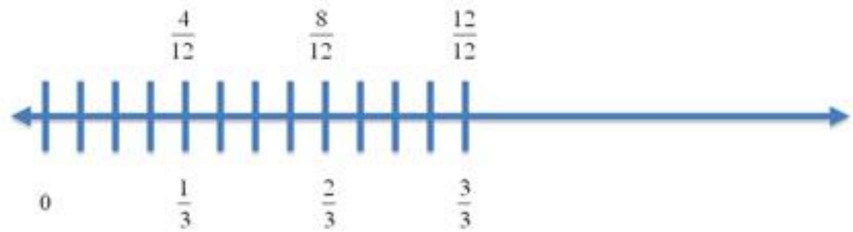
Students will use equivalent fractions as a strategy to add and subtract fractions. Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students apply and extend previous understandings of multiplication and division to multiply and divide fractions. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

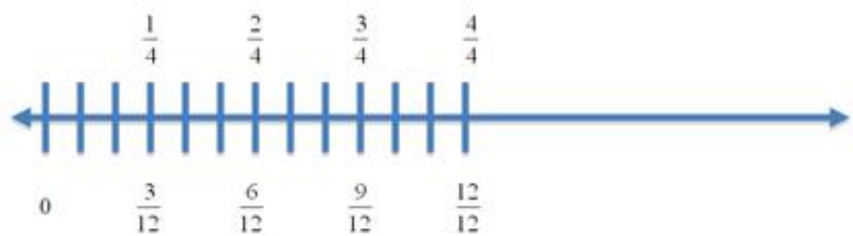
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision are: **fraction, addition/ add, sum, subtraction/subtract, difference, unlike denominator, numerator, benchmark fraction, estimate, reasonableness, mixed numbers, fraction, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional sides lengths, scaling, comparing.**

SCCCR Mathematics Standard	Unpacking What do these standards mean a child will know and be able to do?
5.NSF.1 Add and subtract fractions with unlike denominators (including mixed numbers) using a variety of models, including an area model and a number line.	This standard builds on the work in fourth grade where students add fractions with like denominators. The use of visual fraction models allows students to use reasonableness to find a common denominator prior to using an algorithm. For example, when adding 1/3 + 1/6, Grade 5 students should apply their understanding of equivalent fractions and their ability to rewrite fractions in an equivalent form to find common denominators. <u>Example:</u> **Solve. 1/3 + 1/6** I drew a rectangle and shaded 1/3. I knew that if I cut every third in half then I would have sixths. Based on my picture, 1/3 equals 2/6. Then I shaded in another 1/6. I ended up with an answer of 3/6, which is equal to 1/2.  Step 1 Step 2 Step 3 Step 4 Adding and subtracting fractions with unlike denominators is an area for potential student misconception. As a result, work in 5th grade should build on 3rd grade work where students used a number line to develop an understanding of equivalent fractions (3.NSF.2a). A conceptually based approach for 5th grade addition and subtraction might be to give students two number lines depicting the two fractional addends in a real-world

context (the equivalent fractions on the two number lines must line up). Then ask students to use the number lines to determine a solution. Students will “discover” that the denominators must be the same in order to add/join the two addends. The same problem should then be restated as subtraction and the same number lines used so that students can again make the connection that the denominators must be the same when subtracting.

Another potential area of difficulty for students is how to determine a common denominator where the denominators are not simple multiples of each other. The first step in dealing with unlike denominators would be to rely on students’ 4th grade knowledge (4.NSF.1) where they used the multiplicative identity element (1) in fractional form to determine equivalence. After building on that knowledge they could also multiply the numerator and denominator of each fraction by the denominator of the other to find equivalent fractions that share the same denominator with each other. For example, when adding 1/3 +1/4, students must find a common denominator that can be used to represent both fractions. Students know that the first common multiple of 3 and 4 is 12, so they can draw a number line divided into twelfths.





Using these models, students will see that 1/3 = 4/12 and1/4 = 3/12. Since 4/12 + 3/12 = 7/12, then 1/3 + 1/4 = 7/12

Subtraction of fractions with unlike denominators with area models and number lines is performed in a similar fashion.

5.NSF.2 Solve real-world problems involving addition and subtraction of fractions with unlike denominators.

See 5.NSF.1 for an explanation of how to add and subtract fractions with unlike denominators, using models, to include area models and number lines.

<u>Example:</u>

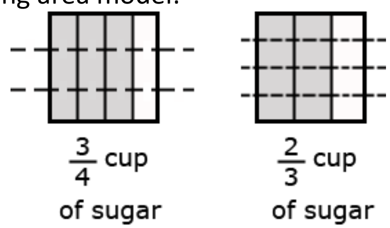
Your teacher gave you 1/4 of the bag of candy. She also gave your friend 1/3 of the bag of candy. If you and your friend combined your candy, what fraction of the bag would you have? Estimate your answer and then solve. How reasonable was your estimate?

<u>Example:</u>

Jerry was making two different types of cookies. One recipe needed 3/4 cup of sugar and the other needed 2/3 cup of sugar. How much sugar did he need to make both recipes?

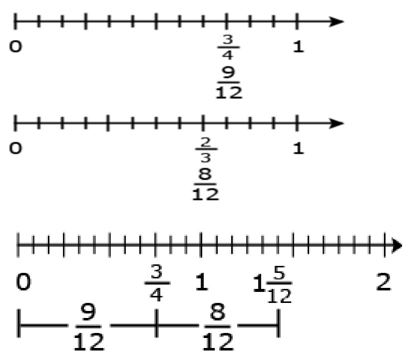
Solution using mental estimation:
A student may say that Jerry needs more than 1 cup of sugar but less than 2 cups. An explanation may compare both fractions to ½ and state that both are larger than ½ so the total must be more than 1. In addition, both fractions are slightly less than 1 so the sum cannot be more than 2.

Solution using area model:



$\frac{3}{4}=\frac{9}{12}$ $\quad$ $\frac{2}{3}=\frac{8}{12}$ $\quad$ $\frac{3}{4}+\frac{2}{3}=\frac{17}{12}=\frac{12}{12}+\frac{5}{12}=1\frac{5}{12}$

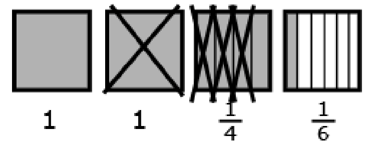
Solution using linear model:



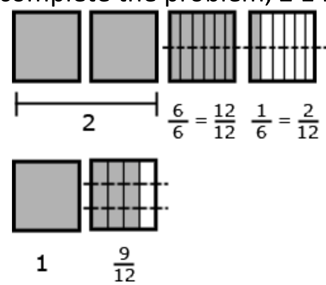
Example:

Solve using a model. 3 1/6 – 1 3/4

This model shows 1 3/4 subtracted from 3 1/6 leaving 1 + ¼ + 1/6 which a student can then change to 1 + 3/12 + 2/12 = 1 5/12.



This diagram models a way to show how 3 1/6 and 1 ¾ can be expressed with a denominator of 12. Once this is accomplished, a student can complete the problem, 2 14/12 – 1 9/12 = 1 5/12.



Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies for calculations with fractions extend from students’ work with whole number operations and can be supported through the use of physical models.

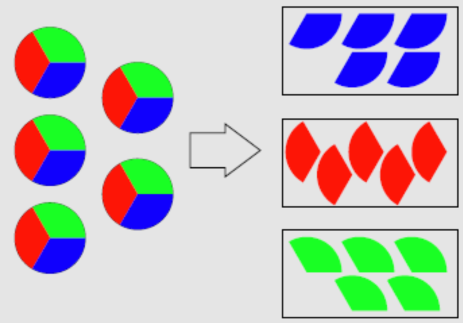
Students make sense of fractional quantities when solving word problems, estimating answers mentally to see if they make sense.

Example:

Cameron and Kellen each have a lemon. They need a cup of lemon juice to make hummus for a party. Cameron squeezes 1/2 a cup from his and Kellen squeezes 2/5 of a cup from his. How much lemon juice do they have? Is it enough?

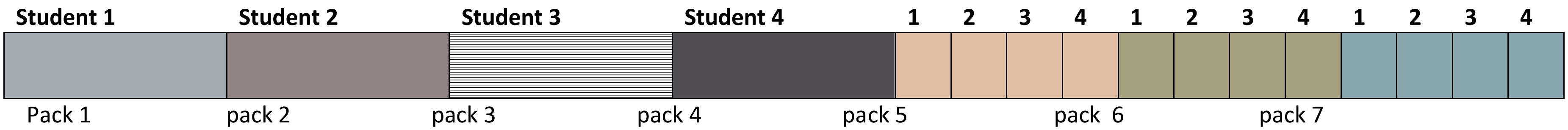
Students estimate that there is almost but not quite one cup of lemon juice, because 2/5 < 1/2. They calculate 1/2 + 2/5 = 9/10, and see this as 1/10 less than 1, which is probably a small enough shortfall that it will not ruin the recipe. They detect an incorrect result such as 2/5 + 2/5 = 3/7 by noticing that 3/7 < 1/2.

(*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 11)

5.NSF.3 Understand the relationship between fractions and division of whole numbers by interpreting a fraction as the numerator divided by the denominator (i.e., $\frac{a}{b} = a \div b$).	Fifth grade students should connect fractions with division, understanding that 5 ÷ 3 = 5/3 Students should explain this by working with their understanding of division as equal sharing. How to share 5 objects equally among 3 shares: $5 \div 3 = 5 \times \frac{1}{3} = \frac{5}{3}$  *If you divide* 5 *objects equally among* 3 *shares, each of the* 5 *objects should contribute* $\frac{1}{3}$ *of itself to each share. Thus each share consists of* 5 *pieces, each of which is* $\frac{1}{3}$ *of an object, and so each share is* $5 \times \frac{1}{3} = \frac{5}{3}$ *of an object.* (*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 11) <u>Examples:</u> **Ten team members are sharing 3 boxes of cookies. How much of a box will each student get?** When working this problem a student should recognize that the 3 boxes are being divided into 10 groups, so he is seeing the solution to the following equation, 10 x n = 3 (10 groups of some amount is 3 boxes) which can also be written as n = 3 ÷ 10. Using models or diagram, they divide each box into 10 groups, resulting in each team member getting 3/10 of a box. **Two afterschool clubs are having pizza parties. For the Math Club, the teacher will order 3 pizzas for every 5 students. For the student council, the teacher will order 5 pizzas for every 8 students. Since you are in both groups, you need to decide which party to attend. How much pizza would you get at each party? If you want to have the most pizza, which party should you attend?** **The six fifth grade classrooms have a total of 27 boxes of pencils. How many boxes will each classroom receive?** Students may recognize this as a whole number division problem but should also express this equal sharing problem as $\frac{27}{6}$. They explain that each classroom gets $\frac{27}{6}$ boxes of pencils and can further determine that each classroom get $4\frac{3}{6}$ or $4\frac{1}{2}$ boxes of pencils.

Example:

Your teacher gives 7 packs of paper to your group of 4 students. If you share the paper equally, how much paper does each student get?



Each student receives 1 whole pack of paper and ¼ of the each of the 3 packs of paper. So each student gets 1 ¾ packs of paper.

5.NSF.4 Extend the concept of multiplication to multiply a fraction or whole number by a fraction.

a. Recognize the relationship between multiplying fractions and finding the areas of rectangles with fractional side lengths;
b. Interpret multiplication of a fraction by a whole number and a whole number by a fraction and compute the product;
c. Interpret multiplication in which both factors are fractions less than one and compute the product.

In earlier grades, students determined the area of rectangles by using area models with square units to fill the rectangle and counting the units, or by multiplying the 2 dimensions to find the total number of square units filling the rectangle. This understanding is extended to rectangles with one or more fractional dimensions. Students will draw the rectangle, filling the rectangle with units to represent the dimensions and add together all the units.

Example:

Mary and Joe determined that the dimensions of their school banner needed to be 4 ft by ¾ ft. What will be the area of the school flag?

A student can draw an array to find this product and can also use his or her understanding of decomposing numbers to explain the multiplication. When drawing the array it is important that students draw to scale in order to see the relationship between the fractional side lengths and the whole number side lengths. In other words, one would not ¼ of a foot the same length as 1 foot in the drawing.

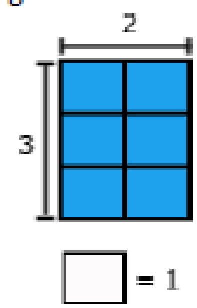
The explanation may include the following:

- First, I am going to multiply ¾ ft by 1 ft, which is ¾ square feet
- I have 4 of these, and ¾ + ¾ + ¾ + ¾ = 12/4
- 12/4 is the same as 3 wholes, so 4 ft x ¾ ft = 3 square feet

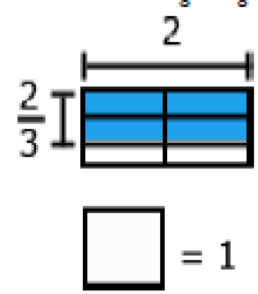
Standard 5.NSF.4 references both the multiplication of a fraction by a whole number and the multiplication of two fractions. Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard.

Example: Building on previous understandings of multiplication

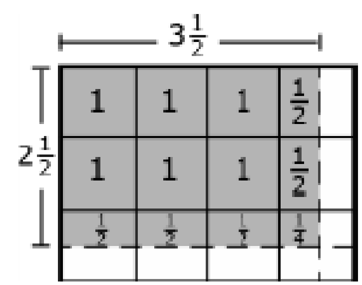
- Rectangle with dimensions of 2 and 3 showing that 2 x 3 = 6.



- Rectangle with dimensions of 2 and $\frac{2}{3}$ showing that $2 \times \frac{2}{3} = \frac{4}{3}$



- $2\frac{1}{2}$ groups of $3\frac{1}{2}$:



(Examples from KATM Grade 5 flip book)

5.NSF.5 Justify the reasonableness of a product when multiplying with fractions. a. Estimate the size of the product based on the size of the two factors; b. Explain why multiplying a given number by a number greater than 1 (e.g., improper fractions, mixed numbers, whole numbers) results in a product larger than the given number; c. Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; d. Explain why multiplying the numerator and denominator by the same number has the same effect as multiplying the fraction by 1.	This standard asks students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine products when multiplying fractions and seeing patterns in outcomes. Students should discover these outcomes rather than being told by the teacher. Using these understandings, students should estimate products before solving problems. a given number x number > 1 = a product > given number Examples: ¾ x 3/2 = 1 1/8 (1 1/8 > ¾) 4 x 1 ½ = 6 (6 > 4) 7/8 x 2 = 1 6/8 (1 6/8 > 7/8) a given number x fraction < 1 = a product < given number Examples: ¾ x ½ = 3/8 (3/8 < ¾) 4 x 1/3 = 1 1/3 (1 1/3 < 4) Example: $2\frac{2}{3}$ x 8 must be more than 8 because 2 groups of 8 is 16 and $2\frac{2}{3}$ is almost 3 groups of 8. So the answer must be close to, but less than 24. Example: Mrs. Bennett is planting two flower beds. The first flower bed is 5 meters long and 2/5 meters wide. The second flower bed is 5 meters long and 5/6 meters wide. How do the areas of these two flower beds compare? Is the value of the area larger or smaller than 5 square meters? Draw pictures to prove your answer.
5.NSF.6 Solve real-world problems involving multiplication of a fraction by a fraction, improper fraction, and a mixed number.	This standard builds on all of the work in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving multiplication by fractions. This standard could include fraction by a fraction, fraction by a mixed number, fraction by improper fraction, improper fraction by mixed number or mixed number by a mixed number

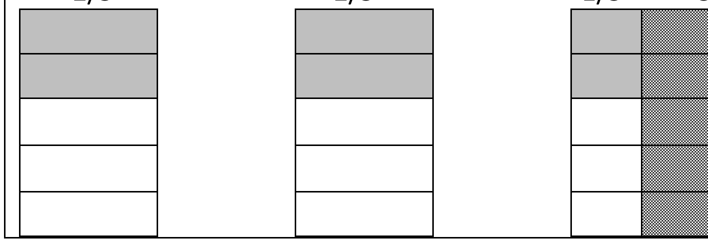
Example:

There are 2 ½ bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. 2/5 of the students that will ride each bus are girls. How many busses would it take to carry *only* the girls?

Student 1

I drew 3 grids and 1 grid represents 1 bus. I cut the third grid in half and I marked out the right half of the third grid, leaving 2 ½ grids. I then cut each grid into fifths, and shaded two-fifths of each grid to represent the number of girls. When I added up the shaded pieces, 2/5 of the 1st and 2nd bus were both shaded, and 1/5 of the last bus was shaded. 2/5 + 2/5 + 1/5 = 5/5 = 1. Therefore, it will take 1 bus to carry only the girls.

2/5 + 2/5 + 1/5 = 5/5 = 1 whole bus.



Student 2

2 ½ x 2/5=

I split the 2 ½ into 2 and ½

2 x 2/5 = 4/5

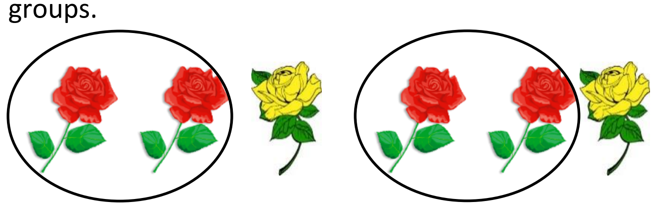
½ x 2/5 = 2/10

I then added 4/5 and 2/10. That equals 1 whole bus load.

Example:

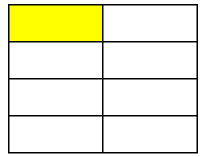
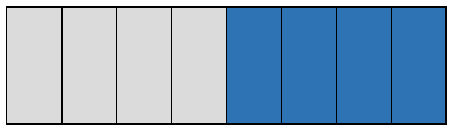
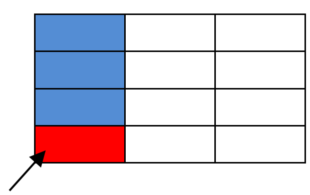
Evan bought 6 roses for his mother. $\frac{2}{3}$ of them were red. How many red roses were there?

Using a visual set model, a student divides the 6 roses into 3 groups and counts how many are in 2 of the 3 groups.



2/3 x 6 = 12/3 = 4

$$\frac{2}{3} \times 6 = \frac{12}{3} = 4 \text{ red roses}$$

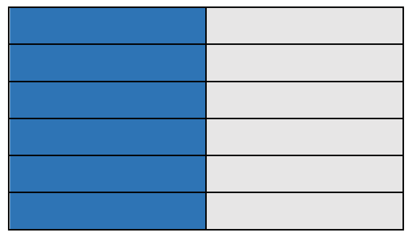
5.NSF.7 Extend the concept of division to divide unit fractions and whole numbers by using visual fraction models and equations. a. Interpret division of a unit fraction by a non-zero whole number and compute the quotient; b. Interpret division of a whole number by a unit fraction and compute the quotient.	This is the first time that students are dividing with fractions. Conceptual understanding is vitally important in order for students to estimate outcomes. In this standard, students divide a unit fraction (fraction with a numerator of 1) by a non-zero whole number. For example ¼ ÷ 2 means that ¼ will be divided into 2 equal parts.  ¼ ÷ 2 = 1/8 Students also divide a whole number by a unit fraction. For example, 2 ÷ ¼ means that 2 wholes will be divided into fourths.  1 whole 1 whole 2 ÷ ¼ = 8 parts Division of a fraction by a fraction is not a requirement at this grade.
5.NSF.8 Solve real-world problems involving division of unit fractions and whole numbers, using visual fraction models and equations.	This standard extends 5.NSF.7 and asks students to solve real-world problems in which they would divide unit fractions by whole numbers or whole numbers by unit fractions. Students should use fraction models and reasoning, and write an equation to represent the solution. <u>Example:</u> **Four students sitting at a table were given 1/3 of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally?** The diagram shows the 1/3 pan divided into 4 equal shares with each share equaling 1/12 of the pan. 1/3 ÷ 4 = 1/12, so each student will receive 1/12 of the pan of brownies. $\frac{1}{3}$ 

Example:

A bowl holds 2 liters of water. If we use a scoop that holds 1/6 of a liter, how many scoops will we need in order to fill the entire bowl?

Possible student explanation:
I created a box to represent the bowl, and divided it into 2 equal parts to represent the 2 liters of water. I then divided each liter of water into sixths to represent the size of the scoop. My answer is the total number of small boxes, which is 12.

$2 \div 1/6 = 12$



Algebraic Thinking and Operations 5.ATO

Overview

Students' work in 5th grade includes evaluating numerical expressions (without variables) involving grouping symbols and translating and interpreting between numerical expressions and verbal phrases. These standards ask students to explain the meanings of the expressions in order to use them to solve problems. Students will generate numerical patterns, translate the patterns into ordered pairs, graph the ordered pairs, and identify the relationship between the patterns.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision are: **evaluate, parentheses, brackets, braces, numerical expressions, expression, grouping symbols, interpret, translate, numerical pattern, rule, ordered pair, coordinate plane, line graph.**

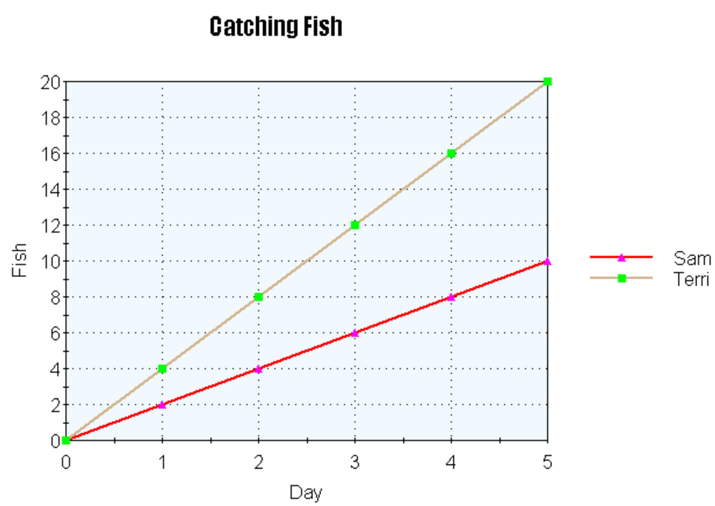
SCCCR Mathematics Standard	**Unpacking** What do these standards mean a child will know and be able to do?
5.ATO.1 Evaluate numerical expressions involving grouping symbols (i.e., parentheses, brackets, braces).	In fifth grade students begin working more formally with expressions. This work should be viewed as an introduction to grouping symbols rather than as an application for order of operations, which is addressed in middle school. The use of the mnemonic phrase "Please Excuse My Dear Aunt Sally" can be misleading and should not be taught. The focus is on introducing students to the symbols (parentheses, brackets, braces). In other words, the expressions included while teaching this standard should use grouping symbols to direct the order in which they are evaluated. Listed below are examples and non-examples. It would be misleading to only use expressions that do not require order of operations. While the focus is on the introduction of symbols, students must understand that there is an "agreed to" order for the operations. This understanding could be derived by giving examples and allowing students to discover that different answers result without an agreed to order. <u>Examples:</u> Students use grouping symbols to evaluate (26 + 18) ÷ 4 12 – {[3+(5-2)] ÷2} (2+3) x (1.5 – 0.5) 6 – (1/2 + 1/3) <u>Non-examples:</u> Students have to apply order of operations to evaluate 26 + 18 ÷4 12 – (3 + 1) ÷2 Multiplication can be indicated with a raised dot, such as 4 · 5, with a raised cross symbol such as 4 × 5, or with parentheses, such as 4(5) or (4)(5). Students need to be exposed to all three notations and should be challenged to understand that all are useful. Begin instruction on this standard using whole numbers and then progress to decimals and fractions. (With decimals choose numbers that are appropriate for using models and drawings) In later grades, "evaluate" means to substitute for a variable and simplify the expression. However at this level students are to only

	simplify the expressions because there are no variables. Mathematically, there cannot be brackets or braces in a problem that does not have parentheses. Likewise, there cannot be braces in a problem that does not have both parentheses and brackets.
5.ATO.2 Translate verbal phrases into numerical expressions and interpret numerical expressions as verbal phrases.	Since students first began to solve story problems in Kindergarten, they related the words in the problem to mathematical symbols, expressions and then equations. For example: "Three bunnies were sitting in the garden and two more joined them. How many bunnies are in the garden?" translates to the expression 3 + 2 and then to the equation 3 + 2 = 5. So, in 5th grade the students are focusing on translating <u>higher level</u> verbal phrases into numerical expressions and building on that to do the reverse – translate numerical expressions as verbal phrases. It should be noted that the emphasis is on expressions without the need to go beyond to an equation. <u>Example:</u> 4(5 + 3) = 32 is an equation. 4(5 + 3) is an expression. The focus of this standard. This standard calls for students to translate between verbal phrases and numerical expressions. <u>Example:</u> **Write an expression for "double five and then add 26."** Student *(2 x 5) + 26* Students write an expression for calculations given in words "such as "divide 144 by 12 and then subtract 7/8. " They write (144 ÷12) – 7/8. Students recognize that 0.5 X (300 / 15) is ½ of (300 ÷ 15) without calculating the quotient. <u>Example:</u> **Describe how the expression 5(10 x 10) relates to 10 x 10.** Student The expression 5(10 x 10) is 5 times larger than the expression 10 x 10 since I know that that 5(10 x 10) means that I have 5 groups of (10 x 10).
5.ATO. 3 Investigate the relationship between two numerical patterns. a. Generate two numerical patterns given two rules and organize in tables;	This standard extends the work from Fourth Grade, where students generate numerical patterns when they are given one rule. In Fifth Grade, students are given two rules and generate two numerical patterns. The graphs that are created should be line graphs to represent the pattern. This is a linear function which is why we get the straight lines.

b. Translate the two numerical patterns into two sets of ordered pairs; c. Graph the two sets of ordered pairs on the same coordinate plane; d. Identify the relationship between the two numerical patterns.	Example: **Describe the pattern: Since Terri catches 4 fish each day, and Sam catches 2 fish, the amount of Terri's fish is always greater. Terri's fish is also always twice as much as Sam's fish. Today, both Sam and Terri have no fish. They both go fishing each day. Sam catches 2 fish each day. Terri catches 4 fish each day. How many fish do they have after each of the five days? Make a graph of the number of fish.** (In the table that follows, the Days are the independent variable, Fish are the dependent variable, and the constant rate is what the rule identifies in the table.) Make a chart (table) to represent the number of fish that Sam and Terri catch.

Days	Sam's Total Number of Fish	Terri's Total Number of Fish
0	0	0
1	2	4
2	4	8
3	6	12
4	8	16
5	10	20

Plot the points on a coordinate plane and make a line graph, and then interpret the graph.



Student Interpretation:
My graph shows that Terri always has more fish than Sam. The number of fish Terri catches increases at a higher rate since she catches 4 fish every day. Sam only catches 2 fish every day, so his number of fish increases at a smaller rate than Terri.

Important to note as well that the lines become increasingly further apart. Identify apparent relationships between corresponding terms. Additional relationships: The two lines will never intersect; there will not be a day in which boys have the same total of fish, explain the relationship between the number of days that has passed and the number of fish a boy has (2n or 4n, n being the number of days).

<u>Example:</u>
Use the rule "add 3" to write a sequence of numbers.
Starting with a 0, students write 0, 3, 6, 9, 12, . . .

Use the rule "add 6" to write a sequence of numbers.
Starting with 0, students write 0, 6, 12, 18, 24, . . .

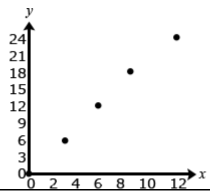
After comparing these two sequences, the students notice that each term in the second sequence is twice the corresponding terms of the first sequence. One way they justify this is by describing the patterns of the terms. Their justification may include some mathematical notation (See example below). A student may explain that both sequences start with zero and to generate each term of the second sequence he/she added 6, which is twice as much as was added to produce the terms in the first sequence. Students may also use the distributive property to describe the relationship between the two numerical patterns by reasoning that 6 + 6 + 6 = 2 (3 + 3 + 3).

0, $^{+3}$ 3, $^{+3}$ 6, $^{+3}$ 9, $^{+3}$12, . . .

0, $^{+6}$ 6, $^{+6}$ 12, $^{+6}$18, $^{+6}$ 24, . . .

Once students can describe that the second sequence of numbers is twice the corresponding terms of the first sequence, the terms can be written in ordered pairs and then graphed on a coordinate grid. They should recognize that each point on the graph represents two quantities in which the second quantity is twice the first quantity.

Ordered Pairs
0,0; 3,6; 6,12;
9,18; 12,24;…

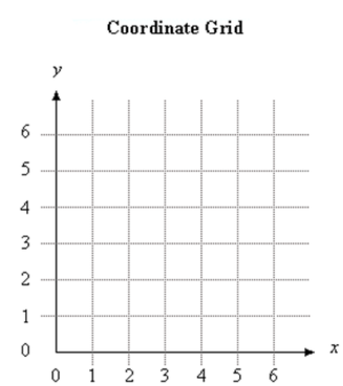


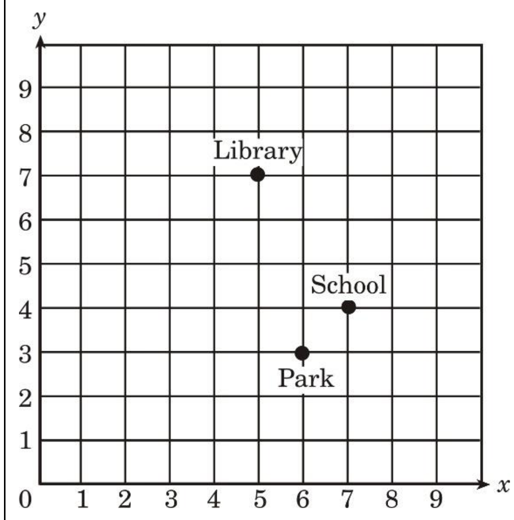
Geometry 5.G

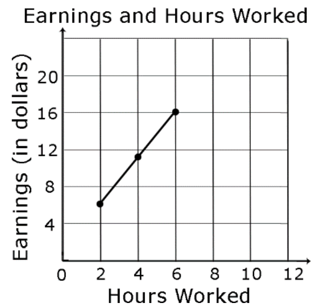
Overview

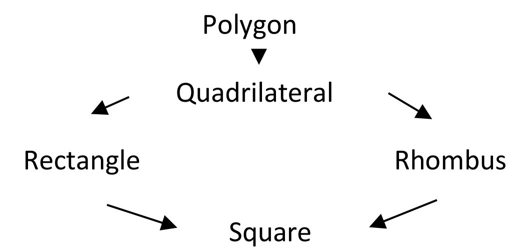
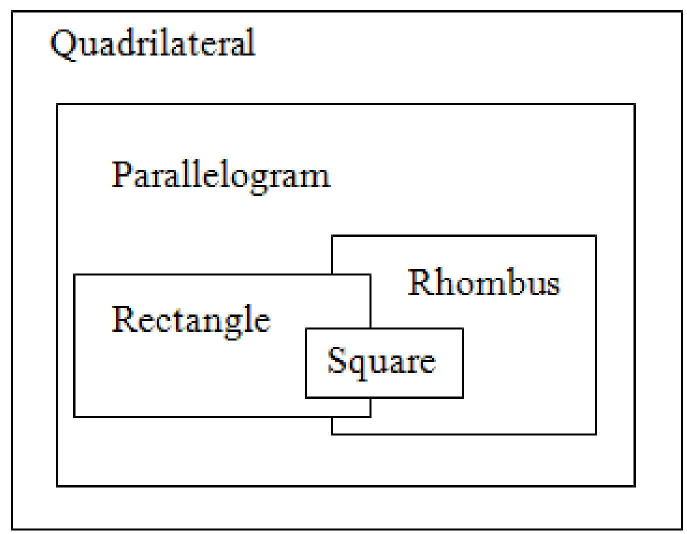
Graph points on the coordinate plane to solve real-world and mathematical problems. Students will classify two-dimensional figures into categories based on their properties.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision are: **coordinate system, coordinate plane, first quadrant, points, lines, axis/axes, x-axis, y-axis, horizontal, vertical, intersection of lines, origin, ordered pairs, coordinates, x-coordinate, y-coordinate, attribute, category, subcategory, hierarchy, (properties)-rules about how numbers work, two dimensional.**
From previous grades: polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, kite
[1]The term "property" in these standards is reserved for those attributes that indicate a relationship between components of shapes. Thus, "having parallel sides" or "having all sides of equal lengths" are properties. "Attributes" and "features" are used interchangeably to indicate any characteristic of a shape, including properties, and other defining characteristics (e.g., straight sides) and nondefining characteristics (e.g., "right-side up").
***(Progressions for the CCSSM, Geometry*, CCSS Writing Team, June 2012, page 3 footnote**

SCCCR Mathematics Standard	**Unpacking** What do these standards mean a child will know and be able to do?
5.G.1 Define a coordinate system. a. The x- and y- axes are perpendicular number lines that intersect at 0 (the origin); b. Any point on the coordinate plane can be represented by its coordinates; c. The first number in an ordered pair is the x-coordinate and represents the horizontal distance from the origin; d. The second number in an ordered pair is the y-coordinate and represents the vertical distance from the origin.	**5.G.1** and **5.G.2** These standards deal with only the first quadrant (positive numbers) in the coordinate plane. Although students can often "locate a point," these understandings are beyond simple skills. For example, initially, students often fail to distinguish between two different ways of viewing the point (2, 3), say, as instructions: "right 2, up 3"; and as the point defined by being a distance 2 from the *y*-axis and a distance 3 from the *x*-axis. In these two descriptions the 2 is first associated with the *x*-axis, then with the *y*-axis. <u>Example:</u> **Connect these points in order on the coordinate grid below:** **(2, 2) (2, 4) (2, 6) (2, 8) (4, 5) (6, 8) (6, 6) (6, 4) and (6, 2). What letter is formed on the grid?**  *Solution: "M" is formed.*

	<u>Example:</u> **Plot these points on a coordinate grid.** **Point A: (2,6)** **Point B: (4,6)** **Point C: (6,3)** **Point D: (2,3)** **Connect the points in order. Make sure to connect Point D back to Point A.** 1. **What geometric figure is formed?** *A trapezoid is formed.* 2. **What line segments in this figure are parallel?** *Line segments AB and DC are parallel.* 3. **What line segments in this figure are perpendicular?** *Line segments AD and DC & line segments AD and AB are perpendicular.* <u>Example:</u> **Emanuel draws a line segment from (1, 3) to (8, 10). He then draws a line segment from (0, 2) to (7, 9). If he wants to draw another line segment that is parallel to those two segments what points will he use?**
5.G.2 Plot and interpret points in the first quadrant of the coordinate plane to represent real-world and mathematical situations.	This standard references real-world and mathematical problems, including the traveling from one point to another and identifying the coordinates of missing points in geometric figures, such as squares, rectangles, and parallelograms. <u>Example:</u> **Using the coordinate grid, which ordered pair represents the location of the School?** **Explain a possible path from the school to the library.** 

	Example: **Sara has saved $20. She earns $8 for each hour she works.** **If Sara saves all of her money, how much will she have after working 3 hours? 5 hours? 10 hours?** **Create a graph that shows the relationship between the hours Sara worked and the amount of money she has saved.** **What other information do you know from analyzing the graph?** Example: **Use the graph below to determine how much money Jack makes after working exactly 9 hours.** 
5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.	This standard calls for students to reason about the attributes (properties) of shapes. Student should have experiences discussing the property of shapes and reasoning. Example: **Examine whether all quadrilaterals have right angles. Give examples and non-examples.** Example: **If the opposite sides on a parallelogram are parallel and congruent, then rectangles are parallelograms.** **A sample of questions that might be posed to students include:** **A parallelogram has 4 sides with both sets of opposite sides parallel. What types of quadrilaterals are parallelograms?** **Regular polygons have all of their sides and angles congruent. Name or draw some regular polygons.** **All rectangles have 4 right angles. Squares have 4 right angles so they are also rectangles. True or False?** **A trapezoid has 2 sides parallel so it must be a parallelogram. True or False?** The notion of congruence ("same size and same shape") may be part of classroom conversation but the concepts of congruence and similarity do **not** appear until middle school.

	<u>TEACHER NOTE:</u> In the U.S., the term “trapezoid” may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with *at least* one pair of parallel sides. The exclusive definition states: **A trapezoid is a quadrilateral with *exactly* one pair of parallel sides.** With this definition, a parallelogram is not a trapezoid. South Carolina has adopted the exclusive definition. (*Progressions for the CCSSM: Geometry*, The Common Core Standards Writing Team, June 2012.) http://illuminations.nctm.org/ActivityDetail.aspx?ID=70
5.G.4 Classify two-dimensional figures in a hierarchy based on their attributes.	<u>Example:</u> Create a Hierarchy Diagram using the following terms: polygon – a closed plane figure formed from line segments that meet only at their endpoints. quadrilateral - a four-sided polygon. rectangle - a quadrilateral with two pairs of congruent parallel sides and four right angles. rhombus – a parallelogram with all four sides equal in length. square – a parallelogram with four congruent sides and four right angles. Possible student solution:  quadrilateral – a four-sided polygon. parallelogram – a quadrilateral with two pairs of parallel and congruent sides. rectangle – a quadrilateral with two pairs of congruent, parallel sides and four right angles. rhombus – a parallelogram with all four sides equal in length. square – a parallelogram with four congruent sides and four right angles. Possible student solution: 

	Student should be able to reason about the attributes of shapes by examining: What are ways to classify triangles? Why can't trapezoids and kites be classified as parallelograms? Which quadrilaterals have opposite angles congruent and why is this true of certain quadrilaterals? and How many lines of symmetry does a regular polygon have? **TEACHER NOTE:** In the U.S., the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with *at least* one pair of parallel sides. The exclusive definition states: **A trapezoid is a quadrilateral with *exactly* one pair of parallel sides.** With this definition, a parallelogram is not a trapezoid. South Carolina has adopted the exclusive definition. (*Progressions for the CCSSM: Geometry*, The Common Core Standards Writing Team, June 2012.)

Measurement and Data 5.MDA

Overview

Students will convert like measurement units within a given measurement system. Students will represent and interpret data using line plots. Students will understand concept of volume and relate volume to multiplication and to addition. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They measure necessary attributes of shapes and use them to solve real-world and mathematical problems, and know when it is appropriate to determine perimeter, area, or volume to solve them.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision are: **conversion/convert, metric and customary measurement.**
From previous grades: relative size, liquid volume, mass, length, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), hour, minute, second, line plot, measurement, attribute, volume, solid figure, right rectangular prism, unit, unit cube, gap, overlap, cubic units (cubic cm, cubic in., cubic ft., nonstandard cubic units), multiplication, addition, edge lengths, height, area of base.

SCCCR Mathematics Standard	Unpacking What do these standards mean a child will know and be able to do?
5.MDA.1 Convert measurements within a single system of measurement: customary (i.e., in., ft., yd., oz., lb., sec., min., hr.) or metric (i.e., mm, cm, m, km, g, kg, mL, L) from a larger to a smaller unit and a smaller to a larger unit.	**5.MDA.1** calls for students to convert measurements within the same system of measurement - customary (standard) or metric. Previous work with conversions includes: 4.MDA.1 Convert measurements within a single system of measurement, customary (i.e., in., ft., yd., oz., lb., sec., min., hr.) or metric (i.e., cm, m, km, g, kg, mL, L) from a larger to a smaller unit. In 5th grade, students also convert from a smaller unit to a larger unit. Examples: 4th Grade: 2 yds = ____ ft 5th Grade: 2 min = ____ sec AND 32 oz = ____ lbs In 3rd grade, students worked primarily with liquid volume, but the customary units of c, pt, qt, gal were converted informally through hands-on measurement activities. Numerical conversions of these units are not included in the standards. In 5th grade, students should review all units included in previous grades before converting them. The only measurement unit new to 5th grade is mm (millimeter). Students should estimate, measure, AND convert using millimeters. Students should explore how the base-ten system supports conversions within the metric system. This is an excellent opportunity to reinforce notions of place value for whole numbers and decimals, and connections between fractions and decimals (e.g., 2 ½ meters can be expressed as 2.5 meters or 250 centimeters). For example, building on the table from Grade 4, Grade 5 students might complete a table of

<table>
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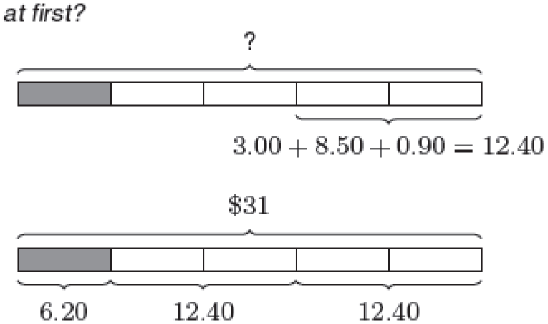
equivalent measurements in feet and inches. Grade 5 students also learn and use such conversions in solving multi-step, real world problems (see example below).

<table>
<tr><th>Feet</th><th>Inches</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td></td><td>1</td></tr>
<tr><td></td><td>2</td></tr>
<tr><td></td><td>3</td></tr>
<tr><td></td><td></td></tr>
</table>

In Grade 6, this table can be discussed in terms of ratios and proportional relationships (see the Ratio and Proportion Progression). In Grade 5, however, the main focus is on arriving at the measurements that generate the table.

Multi-step problem with unit conversion

Kumi spent a fifth of her money on lunch. She then spent half of what remained. She bought a card game for $3, a book for $8.50, and candy for 90 cents. How much money did she have at first?



Students can use tape diagrams to represent problems that involve conversion of units, drawing diagrams of important features and relationships (MP1).

(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, August 2011, page 26)
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<tr>
<td>5. MDA.2 Create a line plot consisting of unit fractions and use operations on fractions to solve problems related to the line plot.</td>
<td>This standard provides a context for students to work with fractions by measuring objects to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot. The line plot should also include mixed numbers in fifth grade.</td>
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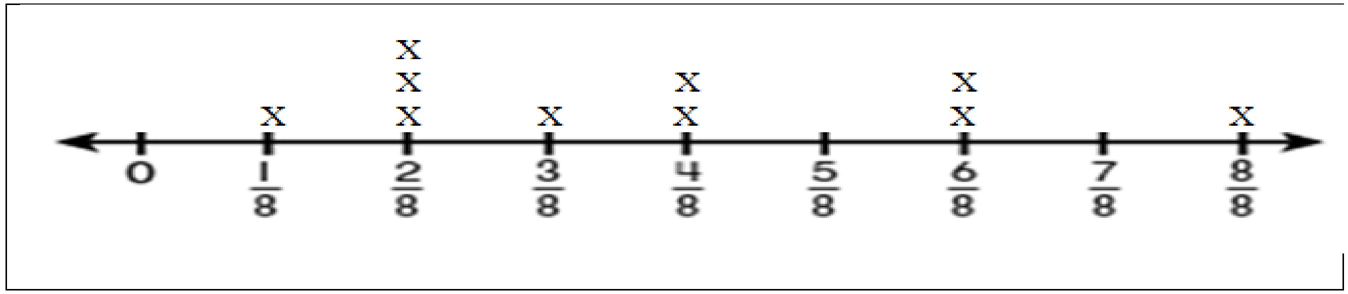
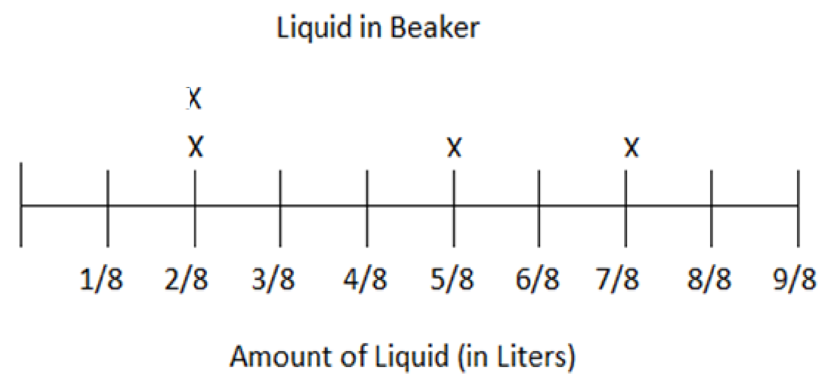
<u>Example:</u>
Students measured objects in their desk to the nearest ½, ¼, or 1/8 of an inch then displayed data collected on a line plot.   How many objects measured ¼ inch? ½inch? If you put all the objects together end to end what would be the total length of all the objects?


<u>Example:</u>
Four beakers, measured in liters, are filled with a liquid.


The line plot above shows the amount of liquid in liters in 4 beakers. If the liquid is redistributed equally, how much liquid would each beaker have?

Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers.
</td>
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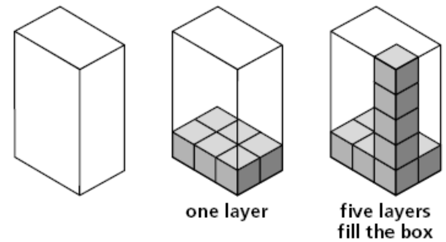
<tr>
<td>5. MDA.3 Understand the concept of volume measurement.</td>
<td>These standards represent the first time that students begin exploring the concept of cubic volume. Students' prior experiences with volume were restricted to liquid volume. In third grade, students began working with area and covering spaces. The concept of volume should be extended from area with the idea that students are</td>
</tr>
</table>

a. Recognize volume as an attribute of right rectangular prisms; b. Relate volume measurement to the operations of multiplication and addition by packing right rectangular prisms and then counting the layers of standard unit cubes; c. Determine the volume of right rectangular prisms using the formula derived from packing right rectangular prisms and counting the layers of standard unit cubes.	covering an area (the bottom face on a cube) with a layer of unit cubes and then adding layers of unit cubes on top of the bottom layer (see picture below). Students should have ample experiences with concrete manipulatives before moving to pictorial representations.  (3 x 2) represented by first layer 6 representing the size/area of one layer (3 x 2) x 5 represented by number of 3 x 2 layers (3 x 2) + (3 x 2) + (3 x 2) + (3 x 2)+ (3 x 2) = 6 + 6 + 6 + 6 + 6 + 6 = 30 As students develop their understanding of volume they learn that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in^3, m^3). Students connect this notation to their understanding of powers of 10 in our place value system. Just as 2^3 is the same as 2 x 2 x 2, m^3 is the same as m x m x m. Thus, students should derive a formula for determining the volume of a rectangular prism, such as Volume of a rectangular prism = the number of cubic units in the first layer x the number of layers, with the measure of the volume always written as cubic units or u^3. Models of cubic inches, cubic centimeters, cubic feet, etc. are helpful in developing an image of a cubic unit. Students might estimate how many cubic yards would be needed to fill the classroom or how many cubic centimeters would be needed to fill a pencil box. (A cubic yard can be constructed using 12 yardsticks.) Volume not only introduces a third dimension and thus a significant challenge to students' spatial structuring, but also complexity in the nature of the materials measured. That is, solid units are "packed," such as cubes in a three-dimensional array, whereas a liquid "fills" three-dimensional space, taking the shape of the container. The unit structure for liquid measurement may be psychologically one dimensional for some students. "Packing" volume is more difficult than iterating a unit to measure length and measuring area by tiling. Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a unit cube. They pack cubes (without gaps) into right rectangular prisms and count the cubes to determine the volume or build right rectangular prisms from cubes and see the layers as they build. They can use the results to compare the volume of right rectangular prisms that have different dimensions. Such experiences enable students to extend their

spatial structuring from two to three dimensions. That is, they learn to both mentally decompose and recompose a right rectangular prism built from cubes into layers, each of which is composed of rows and columns. That is, given the prism, they have to be able to decompose it, understanding that it can be partitioned into layers, and each layer partitioned into rows, and each row into cubes. They also have to be able to compose such as structure, multiplicatively, back into higher units. That is, they eventually learn to conceptualize a layer as a unit that itself is composed of units of units—rows, each row composed of individual cubes—and they iterate that structure. Thus, they might predict the number of cubes that will be needed to fill a box given the net of the box.

Another complexity of volume is the connection between "packing" and "filling." Often, for example, students will respond that a box can be filled with 24 centimeter cubes, or build a structure of 24 cubes, and still think of the 24 as individual, often discrete, not necessarily *units of volume*. They may, for example, not respond confidently and correctly when asked to fill a graduated cylinder marked in cubic centimeters with the amount of liquid that would fill the box. That is, they have not yet connected their ideas about filling volume with those concerning packing volume. Students learn to move between these conceptions, e.g., using the same container, both filling (from a graduated cylinder marked in ml or cc) and packing (with cubes that are each 1 cm^3). Comparing and discussing the volume-units and what they represent can help students learn a general, complete, and interconnected conceptualization of volume as filling three-dimensional space.

Students then learn to determine the volumes of several right rectangular prisms, using cubic centimeters, cubic inches, and cubic feet. With guidance, they learn to increasingly apply multiplicative reasoning to determine volumes, looking for and making use of structure. That is, they understand that multiplying the length times the width of a right rectangular prism can be viewed as determining how many cubes would be in each layer if the prism were packed with or built up from unit cubes. They also learn that the height of the prism tells how many layers would fit in the prism. That is, they understand that volume is a derived attribute that, once a length unit is specified, can be computed as the product of three length measurements or as the product of one area and one length measurement.

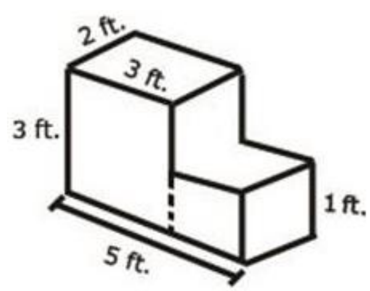
Then, students can learn the formulas $V = l \times w \times h$ and $V = B \times h$ for right rectangular prisms as efficient methods for computing volume, maintaining the connection between these methods and their previous work with computing the number of unit cubes that pack a right rectangular prism.
Note: In the formula $V = B \times h$, B represents the area of the base or length times width.

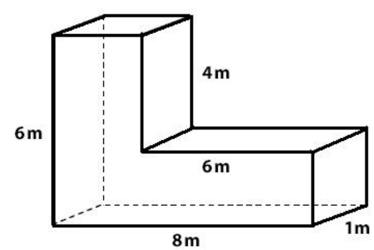
Example:
When given 24 cubes, students make as many rectangular prisms as possible with a volume of 24 cubic units. Students build the prisms and record possible dimensions.

Length	Width	Height
1	2	12
2	2	6
4	2	3
8	3	1

For students who immediately derive the formula for volume, learning experiences should be provided that build on and expand the students' beginning understanding. As in the example below, students may be asked to consider how they can determine volume when a combination of rectangular prisms are used.

Example:
Students determine the volume of concrete needed to build the steps in the diagram below.





5.MDA.4 Differentiate among perimeter, area and volume and identify which application is appropriate for a given situation.	Students leave 5th grade with an understanding of what perimeter, area, and volume are, when to consider them, and how to calculate them. This standard asks students to explain WHEN to use each to solve problems. <u>Examples:</u> **Explain whether you would need to find perimeter, area, or volume to answer the following questions. Explain why you chose each.** • How many feet of wallpaper border are needed for a bedroom wall that is 11 feet long and 9 feet wide? • Mr. Crusoe wants to get the best deal. Which freezer has the most cubic feet for the least amount of money? Freezer A has 25 cubic feet and costs $20 dollars per cubic foot. Freezer B has 16 cubic feet and costs $25 dollars per cubic foot. • A house has a roof with the dimensions of 42ft by 24ft. If plywood comes in pieces that measure 8 feet by 4 feet, how many pieces of plywood are needed to cover the roof? • If you have a plot of land, how many feet of fencing would it take to enclose it? How much corn could you plant on it? • Brenda wants to paint her room. It measures 14 feet x 16 feet x 10 feet. One gallon of paint costs $20 and covers 250 square feet. The paint is sold only in 1 gallon cans. How much will it cost to paint the room? • Mr. Lee wants to build a sandbox 5 feet long, 4 feet wide, and ½ foot high. What length of 6 inch boards will he need to surround the sandbox? How much of his yard will the sandbox cover? How much sand will he need to fill the sandbox? (First decide what you want to know, then put your information into the formula and do the calculations.) • How many square yards of carpet are needed to carpet a room that is 15ft by 25ft? • You have a part-time job at a school. You need to buy enough grass seed to cover the school's soccer field. The field is 50 yards wide and 100 yards long. One bag will cover 5000 square feet. How many bags do you need?

Common Addition and Subtraction Problem Types

<table>
<tr><th></th><th>Result Unknown</th><th>Change Unknown</th><th>Start Unknown</th></tr>
<tr><td>Add to/ Joining</td><td>Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?
2 + 3 = ?</td><td>Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?
2 + ? = 5</td><td>Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?
? + 3 = 5</td></tr>
<tr><td colspan="4">Joining action-involves three quantities; an initial amount, a change amount (the part being added or joined), and the resulting amount (the amount after the action is over).</td></tr>
<tr><td>Take From/ Separating</td><td>Five apples were on the table. I ate two apples. How many apples are on the table now?
5 – 2 = ?</td><td>Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?
5 - ? = 3</td><td>Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?
? – 2 = 3</td></tr>
<tr><td colspan="4">Separation action involves three quantities; the initial amount as the whole or the largest amount, a change, and result amounts.</td></tr>
<tr><th></th><th>Total Unknown</th><th>Addend Unknown</th><th>Both Addends Unknown</th></tr>
<tr><td>Part-Part- Whole</td><td>Three red apples and two green apples are on the table. How many apples are on the table?
3 + 2 = ?</td><td>Five apples are on the table. Three are red and the rest are green. How many apples are green?
3 + ? = 5, 5 – 3 = ?</td><td>Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?
5 = 0 + 5, 5 = 5 + 0
5 = 1 + 4, 5 = 4 + 1
5 = 2 + 3, 5 = 3 + 2</td></tr>
<tr><td colspan="4">Part-Part-Whole action-involves two parts that are combined into one whole. There is no meaningful distinction between the two parts within a part-part-whole situation, so there is no need to have a different problem for each part as the unknown.</td></tr>
<tr><th></th><th>Difference Unknown</th><th>Bigger Unknown</th><th>Smaller Unknown</th></tr>
<tr><td>Compare</td><td>(“How many more?” version):
Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?

(“How many fewer?” version):
Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie?
2 + ? = 5, 5 – 2 = ?</td><td>(Version with “more”):
Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?

(Version with “fewer”):
Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?
2 + 3 = ?, 3 + 2 = ?</td><td>(Version with “more”):
Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?

(Version with “fewer”):
Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have?
5 – 3 = ?, ? + 3 = 5</td></tr>
<tr><td colspan="4">Compare problems involve the comparison of two quantities, and the third amount is the difference between the two amounts. (Adapted from Van de Walle)</td></tr>
</table>

Common Multiplication and Division Problem Types

	Unknown Product 3 x 6 = ?	Group Size Unknown "How many in group?" Division 3 x ? = 18, and 18 ÷ 3 = ?	Number of Groups Unknown "How many groups?" Division ? x 6 = 18, and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? ***Measurement example:*** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? ***Measurement example:*** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? **Measurement example:** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, Area	There are 3 rows of apples with 6 apples in each row. How many apples are there? ***Area example:*** What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? ***Area example:*** A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? ***Area example:*** A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? ***Measurement example:*** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? ***Measurement example:*** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? ***Measurement example:*** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a x b = ?	a x ? = p, and p ÷ a = ?	? x b = p, and p ÷ b = ?

- The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
- The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns. Both forms are valuable. **Harder Array:** The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there?
- Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

The Properties of Operations

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.